**Alone we can do so little;
together we can do so much.**

Helen Keller

Circuit theory Notes

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fb/mna4uonly

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01OFZ Circuit Theory

Academic Year 2014–2015

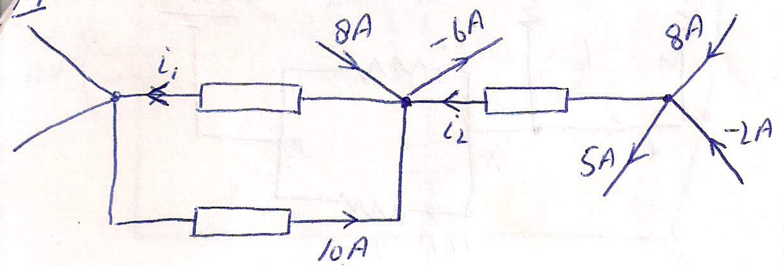
Prof. Stefano Grivet Talocia

$E=mc^2$

$E=mc^2$

CHAPTER-1 FUNDAMENTAL LAWS

1.1



Solution:-

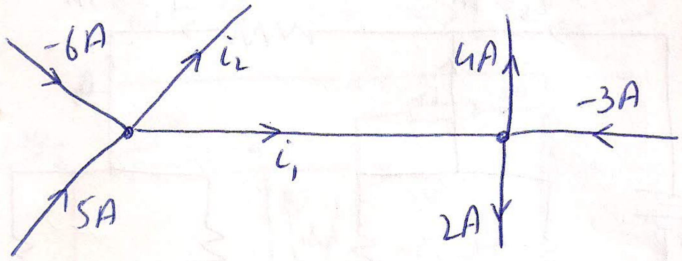
1) $-i_2 + 8 - 2 - 5 = 0$

$-i_2 + 1 = 0 \Rightarrow \boxed{i_2 = 1A}$

2) $-i_1 + 1 + 8 + 6 + 10 = 0$

$-i_1 + 25 = 0 \quad \boxed{i_1 = 25A}$

1.2



Solution:-

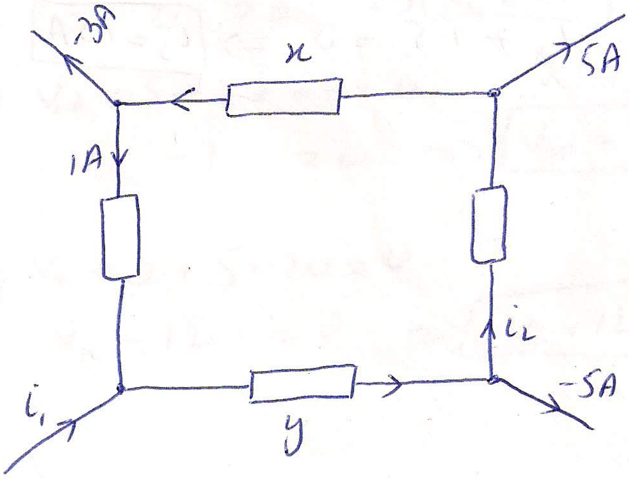
1) $i_1 - 4 - 3 - 2 = 0$

$i_1 - 9 = 0 \quad \boxed{i_1 = 9A}$

2) $-i_2 - 6 + 5 - 9 = 0$

$-i_2 - 10 = 0 \quad \boxed{i_2 = -10A}$

1.3



$i_x + 3 - 1 = 0 \Rightarrow i_x = -2A$

1) $i_2 - 5 + 2 = 0$

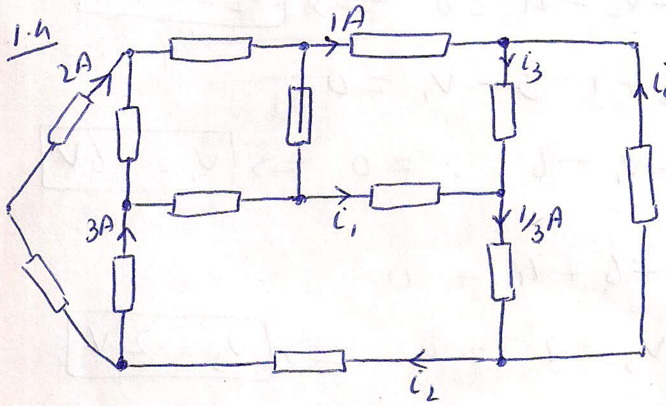
$i_2 - 3 = 0 \Rightarrow \boxed{i_2 = 3A}$

$i_y + 5 - 3 = 0 \Rightarrow i_y = -2A$

2) $i_1 + 2 + 1 = 0$

$i_1 + 3 = 0 \Rightarrow \boxed{i_1 = -3A}$

1.4



1) $i_2 - 2 - 3 = 0$

$i_2 - 5 = 0 \Rightarrow \boxed{i_2 = 5A}$

2) $-5 - i_4 + \frac{1}{3} = 0$

$-i_4 - \frac{14}{3} = 0 \Rightarrow \boxed{i_4 = -\frac{14}{3}A}$

3) $-i_3 + 1 - \frac{14}{3} = 0$

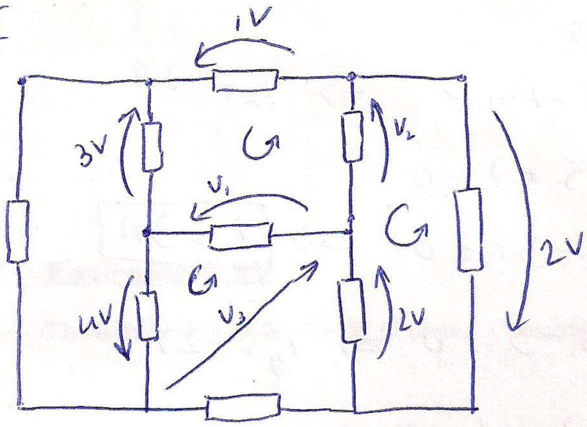
$-i_3 - \frac{11}{3} = 0 \Rightarrow \boxed{i_3 = -\frac{11}{3}A}$

4) $i_1 - \frac{11}{3} - \frac{1}{3} = 0$

$i_1 - \frac{12}{3} = 0$

$\boxed{i_1 = 4A}$

1.5



Solution

1) $-V_2 - 2 - 2 = 0$

$-V_2 - 4 = 0 \Rightarrow \boxed{V_2 = -4V}$

2) $-4 + 1 - 3 - V_1 = 0$

$-V_1 - 6 = 0 \Rightarrow \boxed{V_1 = -6V}$

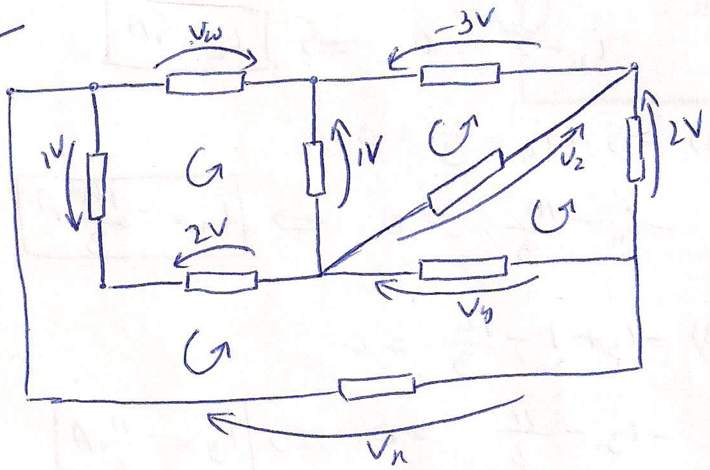
3) $V_3 - 6 + 4 = 0$

$V_3 - 2 = 0 \Rightarrow \boxed{V_3 = 2V}$

——— × ———

4)

1.6



Solution 1) $-V_w + 1 - 2 + 1 = 0$

$\boxed{V_w = 0V}$

2) $V_z - 3 - 1 = 0$

$V_z - 4 = 0 \Rightarrow \boxed{V_z = 4V}$

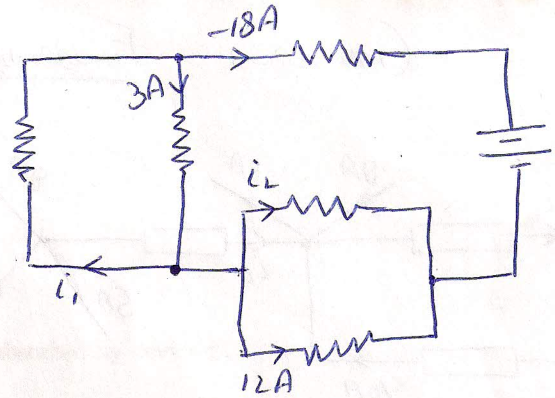
3) $-V_y + 2 - 4 = 0$

$-V_y - 2 = 0 \Rightarrow \boxed{V_y = -2V}$

4) $-V_x + 2 - 3 = 0$

$-V_x - 1 = 0 \Rightarrow \boxed{V_x = -1V}$

1.7



Solution 1) $i_1 - 3 + 18 = 0$

$i_1 + 15 = 0 \Rightarrow \boxed{i_1 = -15A}$

2) $\cancel{i_2 + 3 + 15 = 0}$

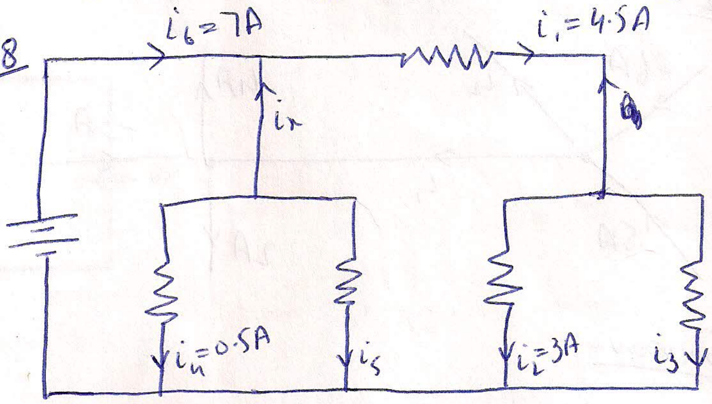
$\cancel{-i_L + 18 = 0 \Rightarrow i_2}$

$i_L + 12 - 18 = 0$

$i_L - 6 = 0 \Rightarrow \boxed{i_L = 6A}$

——— × ———

1.8



1) $i_x + 7 - 4.5 = 0$

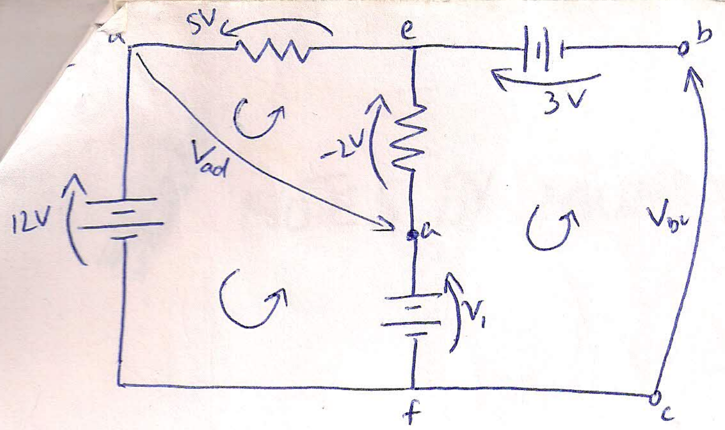
$i_x + 2.5 = 0 \Rightarrow i_x = -2.5A$

$-i_5 + 2.5 - 0.5 = 0$

$-i_5 + 2 = 0 \Rightarrow \boxed{i_5 = 2A}$

2) $-i_3 - 3 + 4.5 = 0$

$-i_3 + 1.5 = 0 \Rightarrow \boxed{i_3 = 1.5A}$



Solution

1) $V_{ad} - 2 + 5 = 0$

$V_{ad} + 3 = 0 \Rightarrow \boxed{V_{ad} = -3V}$

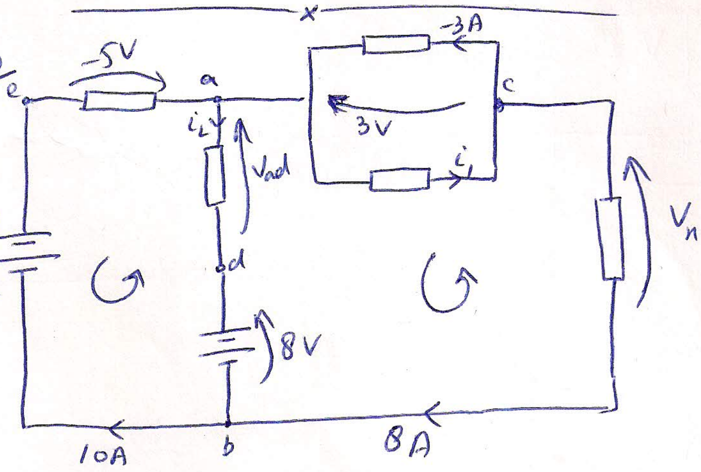
2) $V_1 - 2 + 5 - 12 = 0$

$V_1 - 9 = 0 \Rightarrow \boxed{V_1 = 9V}$

3) $V_{bc} + 3 + 2 - 9 = 0$

$V_{bc} - 4 = 0 \Rightarrow \boxed{V_{bc} = 4V}$

1.10



Solution 1) $i_1 + 3 - 8 = 0$

$i_1 - 5 = 0 \Rightarrow \boxed{i_1 = 5A}$

2) $i_2 + 8 - 10 = 0$

$i_2 - 2 = 0 \Rightarrow \boxed{i_2 = 2A}$

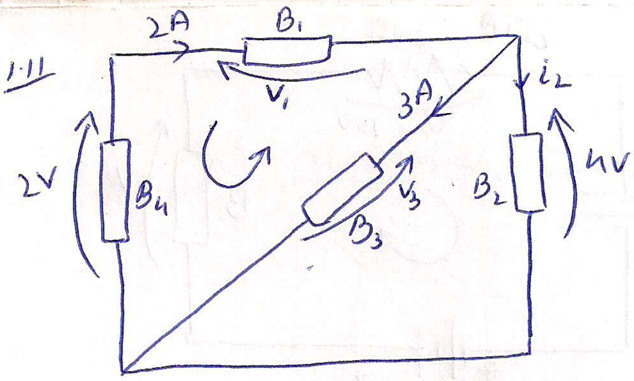
3) $V_{ad} + 5 - 20 + 8 = 0$

$V_{ad} - 7 = 0 \Rightarrow \boxed{V_{ad} = 7V}$

4) $V_x + 3 + 5 - 20 = 0$

$V_x - 12 = 0 \Rightarrow \boxed{V_x = 12V}$

1.11



Solutions $-i_2 - 3 + 2 = 0$

$\boxed{i_2 = -1A}$

$V_1 - 2 + 4 = 0$

$\boxed{V_1 = -2V}$

$\boxed{V_3 = 4V}$

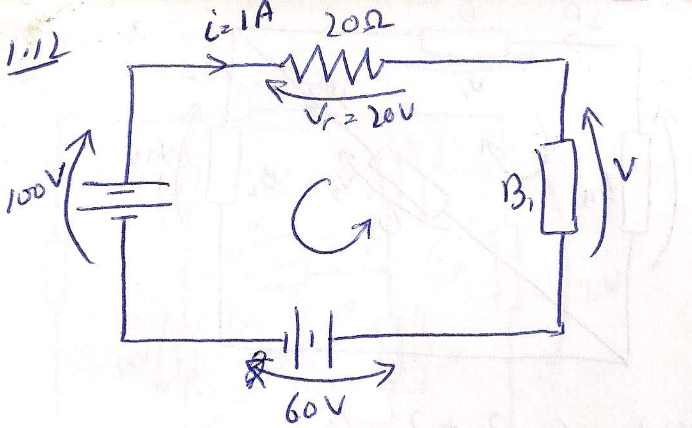
1) $P_1 = V_1 i_1$
$= 2(-2) = -4W$

2) $P_2 = V_2 i_2$
$= 4(-1) = -4W$

3) $P_3 = V_3 i_3$
$= 4(3) = 12W$

4) $P_4 = V_4 i_4$
$= 2(-2) = -4W$

1.12



As $P = Vi$

$i = \frac{100}{100} = 1A \Rightarrow \boxed{i = 1A}$

$V_r = 20(1) = 20V$

$V + 20 - 100 + 60 = 0$

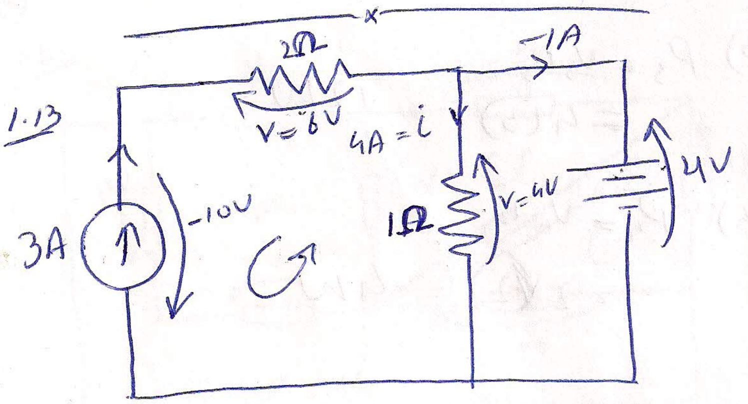
$V - 20 = 0$

$\boxed{V_1 = 20V}$

$P_1 = V_1 i$

$P_1 = 20(1) \Rightarrow \boxed{P_1 = 20W}$

1.13



$i = 4A$

$V_r = 6V$

$V_I + 4 + 6 = 0$

$V_I = -10V$

$P_1(E = 4V) = 4(-1) = -4W$

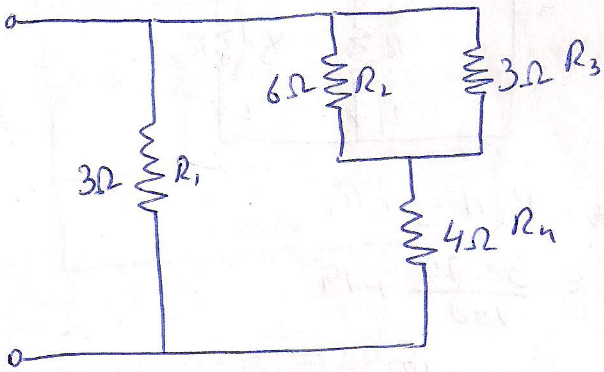
$P(R = 1\Omega) = 4(4) = 16W$

$P(R = 2\Omega) = 6(3) = 18W$

$P(I = 3A) = -10(3) = -30W$

Chapter-2 Resistive Circuits

Compute resistance from A & B.



Solution-

$$((R_2 \parallel R_3) + R_4) \parallel R_1$$

$$= \left(\frac{6.3}{6+3} + 4\right) \parallel R_1$$

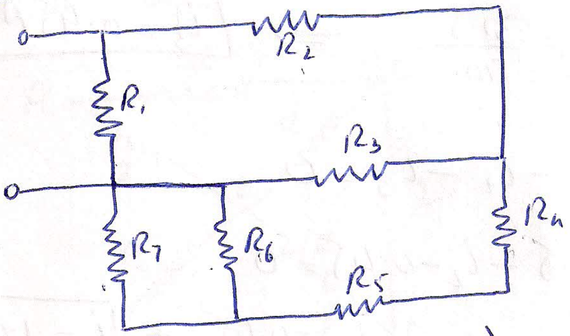
$$= \left(\frac{18}{9} + 4\right) \parallel R_1$$

$$= \frac{6.3}{6+3} = \frac{18}{9} = 2\,\Omega$$

$R_{eq} = 2\,\Omega$ Ans

x

2.2



$$((R_7 \parallel R_6 + R_5 + R_4) \parallel R_3 + R_2) \parallel R_1$$

$$R_{eq} = \left(\frac{1000.\cancel{1000}}{2\cancel{000}} + 1000 + 1000 \parallel R_3 + R_2\right) \parallel R_1$$

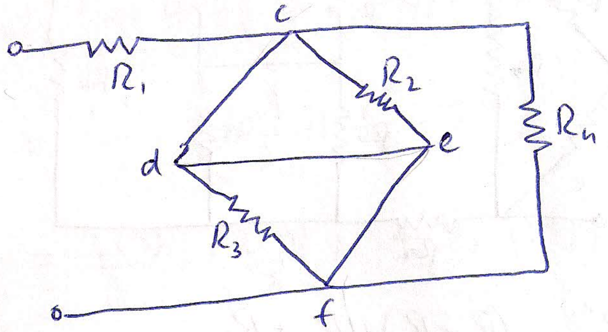
$$R_{eq} = \frac{2500.\cancel{1000}}{3\cancel{000}} = \frac{25000}{35}$$

$$= (714.285 + 1000) \parallel R_1$$

$$= \frac{1714.285 \times 1000}{2714.285}$$

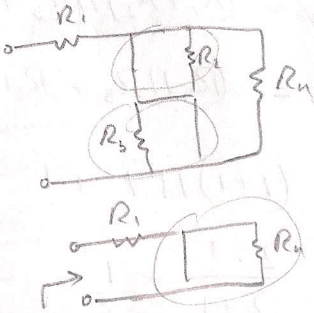
$$\boxed{R_{eq} = 631.57\,\Omega}$$

2.3



$R_{eq} = R_1$

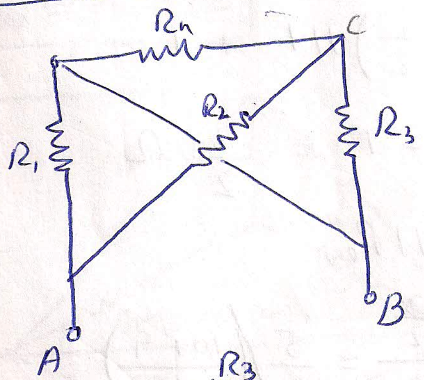
$$\boxed{R_{eq} = 15\,\Omega}$$

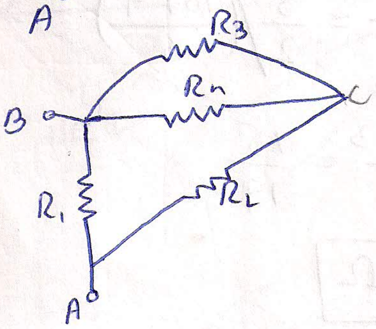


$R_{eq} = R_1$

x

2.4





$$R_{eq} = (R_3 \parallel R_4 + R_2) \parallel R_1$$

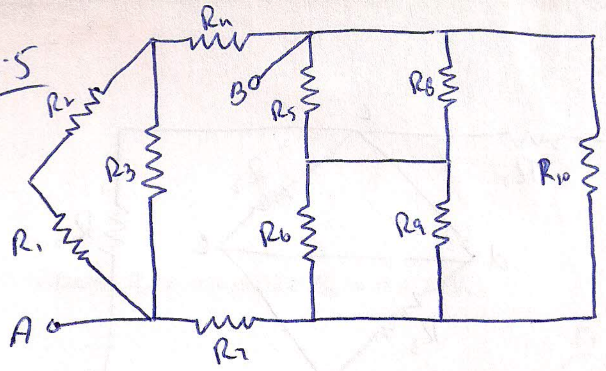
$$R_{eq} = \left(\frac{40.4\cancel{0}}{8\cancel{0}} + 20\right) \parallel R_1$$

$$R_{eq} = (20 + 20) \parallel R_1$$

$$= \frac{40.1\cancel{0}}{5\cancel{0}}$$

$$\boxed{R_{eq} = 8\,\Omega}$$

2.5



1) $R'_{eq} = (R_1 + R_2) \parallel R_3 + R_4$

$R''_{eq} = (R_5 \parallel R_8 + R_6 \parallel R_9) \parallel R_{10} + R_7$

$R'_{eq} = (1+1) \parallel 1 + 1$

$= \frac{2 \cdot 1}{2+1} + 1$

$= \frac{2}{3} + 1 = \frac{5}{3}\,\Omega$

$R''_{eq} = \left(\frac{1}{2} + \frac{1}{2}\right) \parallel 1 + 1$

$= \frac{1}{2} + 1 = \frac{3}{2}\,\Omega$

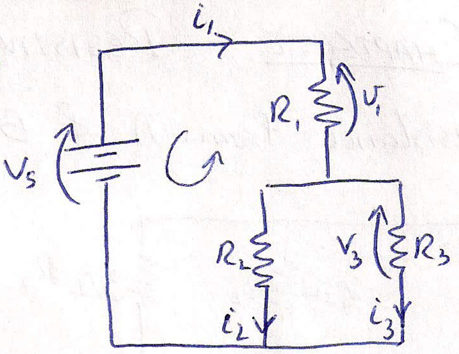
$R_{eq} = R'_{eq} \parallel R''_{eq}$

$= \frac{\frac{5}{\cancel{3}} \cdot \frac{\cancel{3}}{2}}{\frac{5}{3} + \frac{3}{2}} = \frac{5}{2} \Big/ \left(\frac{10+9}{6}\right)$

$= \frac{5}{\cancel{2}} \times \frac{\cancel{6}^{3}}{19} = \frac{15}{19}$

$\boxed{R_{eq} = 0.789\,\Omega}$

2.10



$R_{eq} = R_2 \parallel R_3 + R_1$

$= \frac{30 \cdot 70}{100} + 19$

$R_{eq} = 21 + 19 = 40\,\Omega$

$V_s = i_1 R_{eq}$

$i_1 = \frac{60}{40} \Longrightarrow \boxed{i_1 = 1.5\,A}$

$V_1 = i_1 R_1$

$V_1 = (1.5)19 \Rightarrow \boxed{V_1 = 28.5\,V}$

$-V_s + V_3 + V_1 = 0$

$-60 + V_3 + 28.5 = 0 \Rightarrow \boxed{V_3 = 31.5\,V}$

$V_3 = i_3 R_3$

$i_3 = \frac{31.5}{70} \Longrightarrow \boxed{i_3 = 0.45\,A}$

$i_1 - i_2 - i_3 = 0$

$1.5 - i_2 - 0.45 = 0$

$i_2 = 1.5 - 0.45 \Rightarrow \boxed{i_2 = 1.05\,A}$

$V_1 = \frac{19}{\cancel{40}_{2}} \times \cancel{60}^{3} = 28.5\,V$

$V_3 = \frac{21}{40} \times 60 = 31.5\,V$

$V_s = i_1 R_{eq} \Rightarrow i_1 = 1.5\,A$

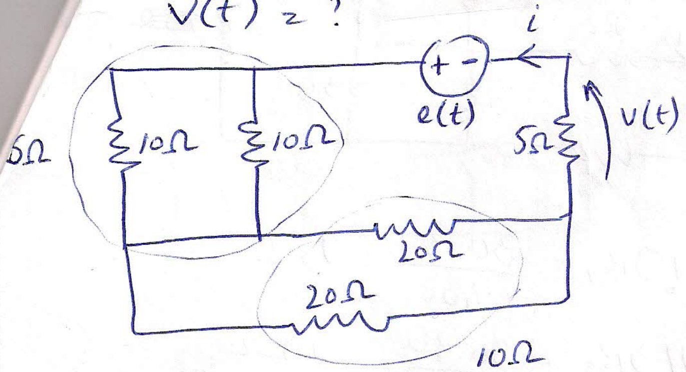
$i_2 = \frac{70}{100} \times 1.5 \Rightarrow i_2 = 1.05\,A$

$i_3 = \frac{30}{100} \times 1.5 \Rightarrow i_3 = 0.45\,A$

P2

$e(t) = E_0 \sin(\omega t + \varphi)$

$v(t) = ?$



$R_{eq} = 5 + 10 + 5 = 20\Omega$

$i = \dfrac{e(t)}{R_{eq}}$

$i = \dfrac{E_0 \sin(\omega t + \varphi)}{20}$

$v(t) = i \cdot R$

$= \dfrac{5\, E_0 \sin(\omega t + \varphi)}{20}$

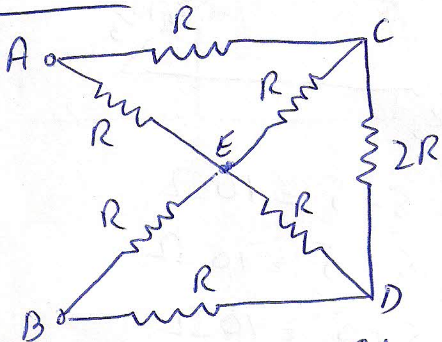
$v(t) = \dfrac{-5}{20} e(t)$

$\boxed{v(t) = -0.25\, E_0 \sin(\omega t + \varphi)}$

$v(t) = -0.25\, E_0 \sin(\omega t + \varphi)$

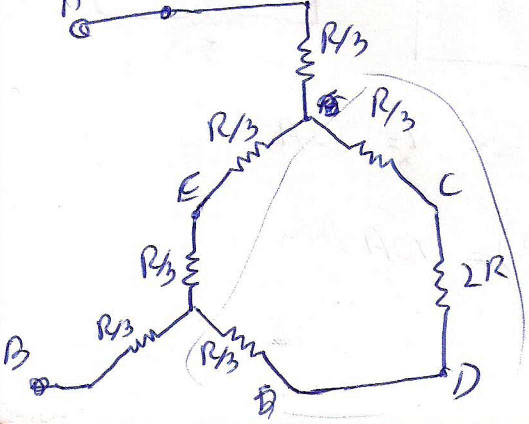
(-ve) is due to the same direction of i & $v(t)$.

2.7



Transform from delta to star.

$r = \dfrac{R}{3}$



$R_{eq} = \dfrac{R}{3} + \dfrac{R}{3} + \left[\left(\dfrac{R}{3} + \dfrac{R}{3}\right) \parallel \left(\dfrac{R}{3} + 2R + \dfrac{R}{3}\right)\right]$

$= \dfrac{2R}{3} + \left(\dfrac{2R}{3} \parallel \dfrac{2R}{3} + 2R\right)$

$= \dfrac{2(9)}{3} + \left(\dfrac{2(9)}{3} \parallel \dfrac{2(9)}{3} + 2(9)\right)$

$= 6 + (6 \parallel 24)$

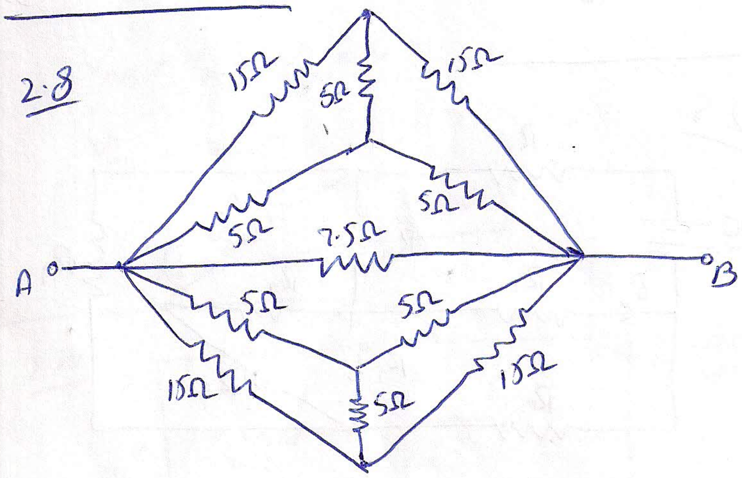
$= 6 + \dfrac{6 \cdot 24}{30}$

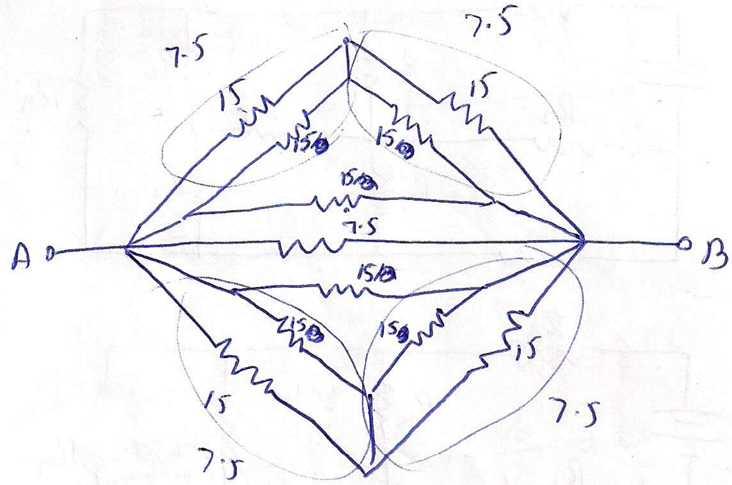
$= 6 + \dfrac{144}{30}$

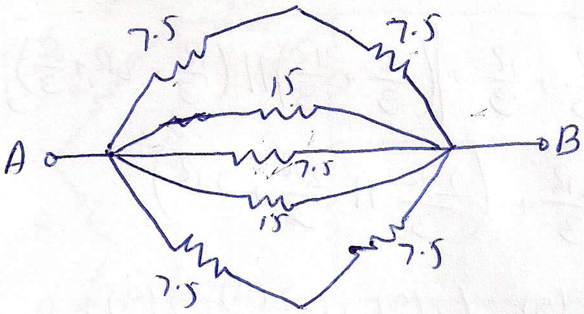
$= \dfrac{180 + 144}{30} = \dfrac{324}{30}$

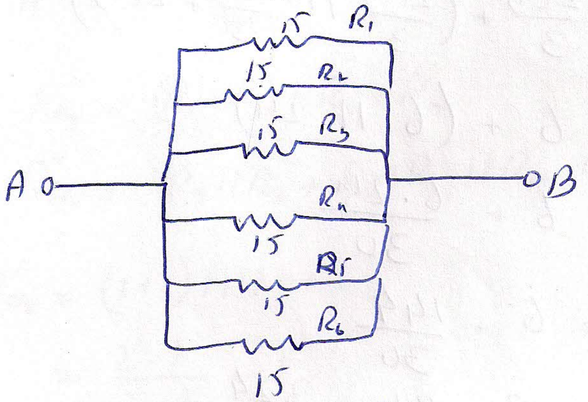
$\boxed{R_{eq} = 10.8\,\Omega}$

2.8





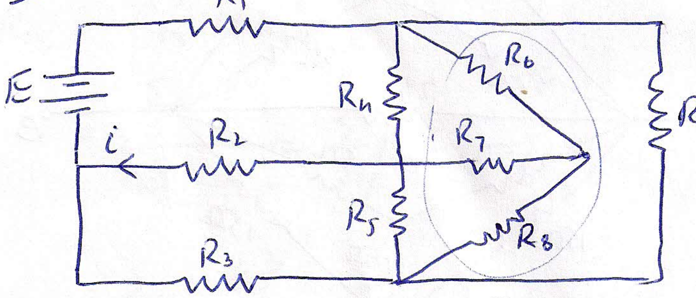




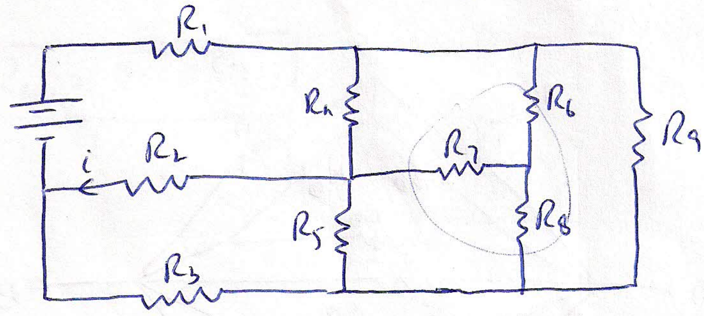
$$R_{eq} = \frac{R}{6} \qquad \left(R_1 = R_2 = R_3 = R_4 = R_5 = R_6\right)$$

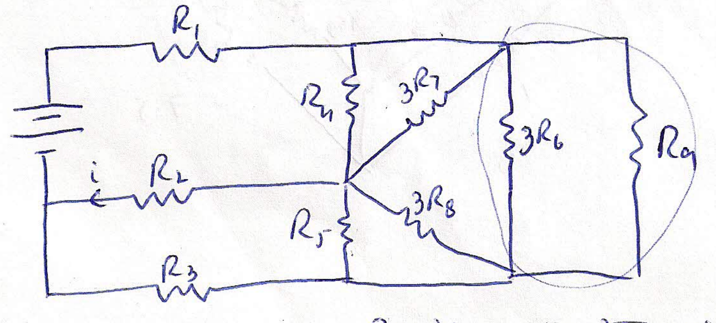
$$= \frac{15}{6} = 2.5\,\Omega$$

2.9



star

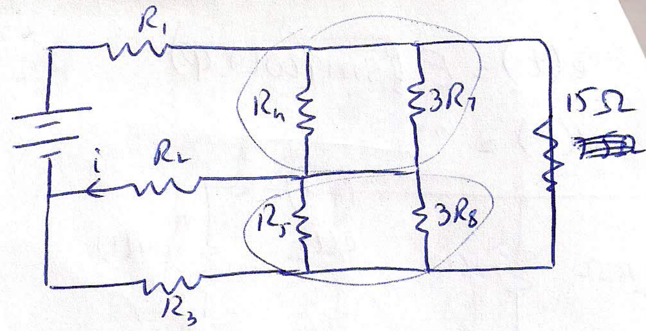




$\frac{30.30}{60}$

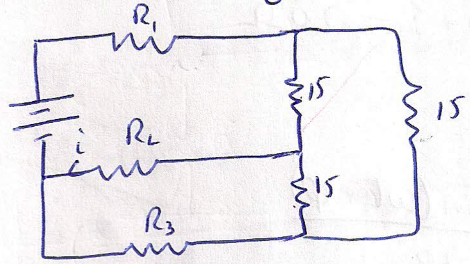
$\frac{900}{60} = \frac{30}{2} = 15$

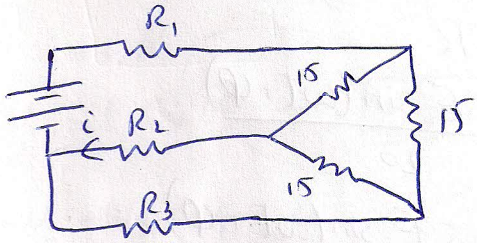
$\frac{10.30}{40} = \frac{300}{40} = 7.5\,\Omega$

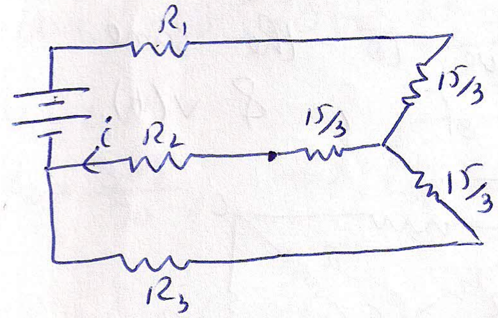


$$R_4 \parallel 3R_7 = \frac{30.30}{60} = 15\,\Omega$$

$$R_5 \parallel 3R_8 = \frac{30.30}{60} = 15\,\Omega$$



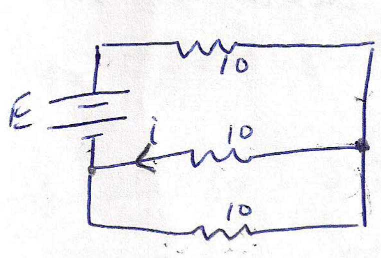


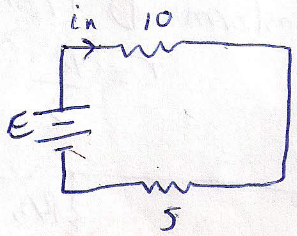


$$R_3 + 5 = 5 + 5 = 10\,\Omega$$

$$R_2 + 5 = 5 + 5 = 10\,\Omega$$

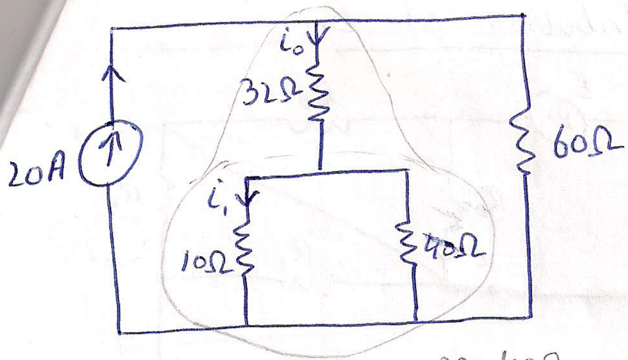
$$R_1 + 5 = 5 + 5 = 10\,\Omega$$





$$i_n = \frac{300}{10+5} \Rightarrow i_n = 20A$$

$$\text{so } i(R = 10\,\Omega) = 10A$$



$R_{eq} = 8\Omega + 32 = 40\Omega$

$$i_o = \frac{60}{40+60} \times 20$$

$$= \frac{12\cancel{00}}{1\cancel{00}} \Rightarrow i_o = 12A$$

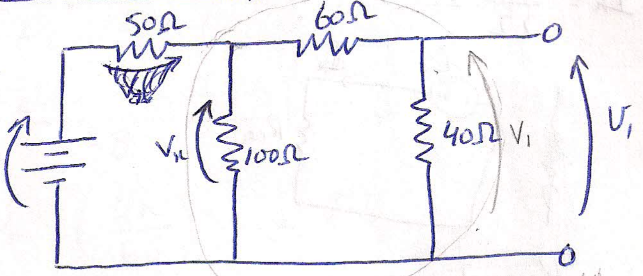
Now

$$i_1 = \frac{40}{10+40} \times 12$$

$$= \frac{4\cancel{0}}{5\cancel{0}} \times 12$$

$$i_1 = \frac{48}{5} \Rightarrow \boxed{i_1 = 9.6A}$$

2.12



$R_{eq} = 60 + 40 \| 100$

$R_{eq} = \frac{100.100}{200} = 50\Omega$

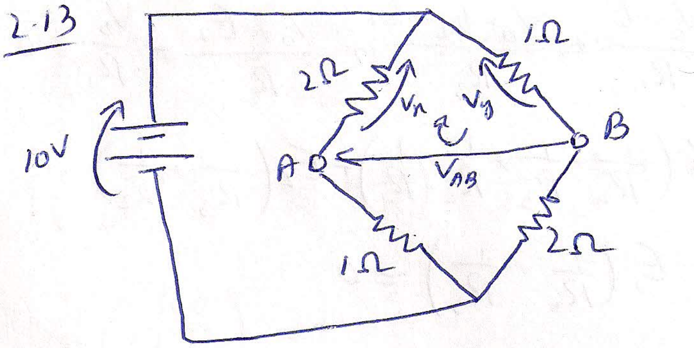
$\frac{100.60}{160}$

$$V_x = \frac{50}{50+50} \times 100 = 50V$$

$$V_1 = \frac{40}{60+40} \times 50$$

$$= \frac{20\cancel{00}}{1\cancel{00}} \Rightarrow \boxed{V_1 = 20V}$$

2.13



$$V_x = \frac{2}{2+1} \times 10$$

$$V_x = \frac{2}{3} \times 10 \Rightarrow V_x = 6.66V$$

$$V_y = \frac{1}{2+1} \times \cancel{6.66}\,10$$

$$= \frac{1}{3} \times \cancel{6.66}\,10 \Rightarrow \cancel{V_y = 2.22V}$$

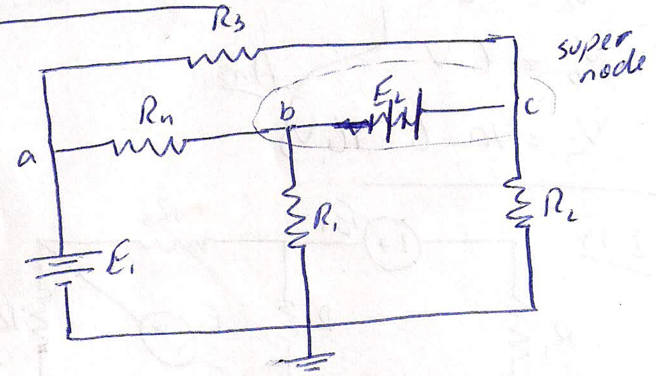
$$V_y = 3.33V$$

$$V_{AB} + V_x - V_y = 0$$

$$V_{AB} + 6.66 - 3.33 = 0$$

$$\boxed{V_{AB} = -3.33V}$$

2.14



$$V_a = E_1$$

$$V_c = E_2 + V_b$$

Taking super node apply KCL

$$\frac{V_b - V_a}{R_4} + \frac{V_c - V_a}{R_3} + \frac{V_b}{R_1} + \frac{V_c}{R_2} = 0$$

$$\frac{V_b - E_1}{R_4} + \frac{E_2 + V_b - E_1}{R_3} + \frac{V_b}{R_1} + \frac{E_2 + V_b}{R_2} = 0$$

$$\frac{V_b-E_1}{R_4}+\frac{V_b+E_2-E_1}{R_3}+\frac{V_b+E_2}{R_2}+\frac{V_b}{R_1}=0$$

$$V_b\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_1}\right)+E_2\left(\frac{1}{R_3}+\frac{1}{R_2}\right)$$

$$-E_1\left(\frac{1}{R_4}+\frac{1}{R_3}\right)=0$$

$$V_b\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{8}+\frac{1}{12}\right)+10\left(\frac{1}{8}+\frac{1}{8}\right)-$$

$$12\left(\frac{1}{2}+\frac{1}{8}\right)=0$$

$$V_b\left(\frac{9+1}{12}\right)+\cancel{10}^{5}\left(\frac{1}{\cancel{4}_{2}}\right)-\cancel{12}^{3}\left(\frac{5}{\cancel{8}_{2}}\right)=0$$

$$V_b\left(\frac{10}{12}\right)+\frac{5}{2}-\frac{15}{2}=0$$

$$V_b\left(\frac{\cancel{B}5}{6}\right)+\frac{5-15}{2}=0$$

$$V_b\,\frac{5}{6}=\frac{\cancel{10}^{5}}{\cancel{2}}$$

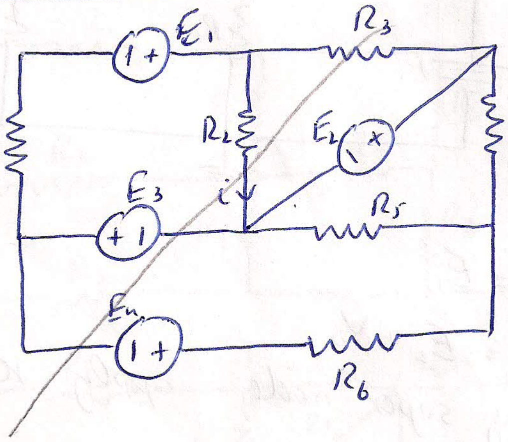
$$V_b=\cancel{5}\times\frac{6}{\cancel{5}}$$

$$V_b=6V$$

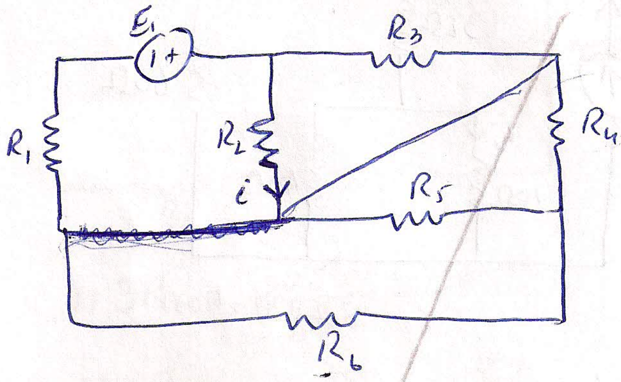
$V_a=12V$
$V_b=6V$
$V_c=10+6=16V$

Ans

2.15

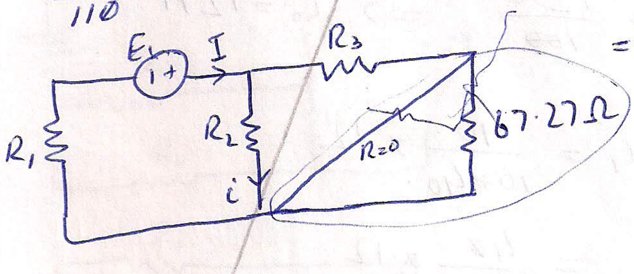


Contribution of E_1



$R_5 \parallel R_6+R_4$

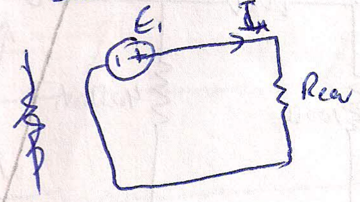
$$\frac{50\cdot 60}{110}+40=67.27\Omega$$



$R_2 \parallel R_3+R_1$

$$R_{eq}=\frac{20\cdot 30}{50}+10=\cancel{8}12+10=22\Omega$$

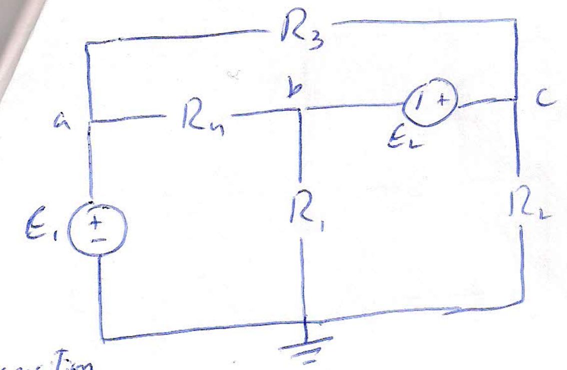
$$I_x=\frac{10}{22}\Longrightarrow i_x=0.45A$$



$$i_1=\frac{R_3}{R_2+R_3}I$$

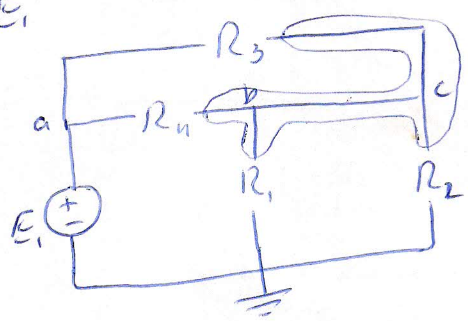
$$=\frac{30}{50}\times 0.45$$

$$i_1=0.27A$$



Superposition

E_1



$V'_a = E_1 \qquad V'_b = V'_c$

$$\frac{V'_b - E_1}{R_4} + \frac{V'_b}{R_1} + \frac{V'_b}{R_2} + \frac{V'_b - E_1}{R_3} = 0$$

$$V'_b\left(\frac{1}{2} + \frac{1}{12} + \frac{1}{8} + \frac{1}{8}\right) - \frac{12}{2} - \frac{3}{2} = 0$$

$$V'_b\left(\frac{24 + 4 + 6 + 6}{48}\right) = 6 + \frac{3}{2}$$

$$V'_b\left(\frac{40}{48}\right) = \frac{\cancel{6}\,15}{2}$$

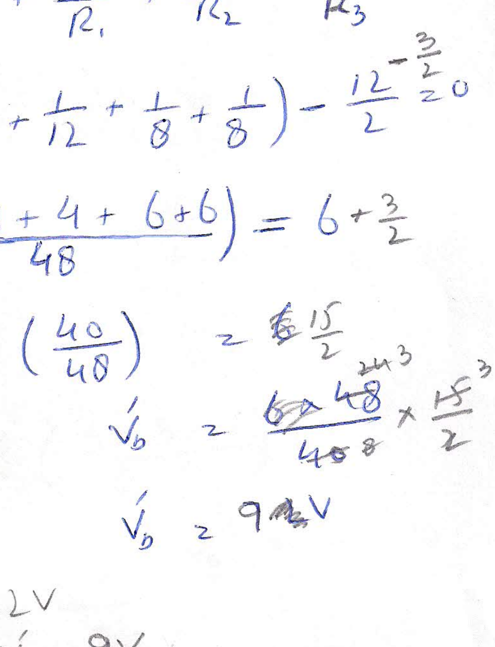
$$V'_b = \frac{\cancel{6\times} \cancel{48}^{\,\cancel{24}\,3}}{\cancel{40}\,8} \times \frac{\cancel{15}^{\,3}}{2}$$

$$V'_b = 9\,V$$

$V'_a = 12\,V$

$V'_b = V'_c = 9\,V$

E_2



$V''_a = 0 \quad , \quad V''_c = V''_b + E_2$

$$\frac{V''_b}{R_4} + \frac{V''_b}{R_1} + \frac{V''_b + 10}{R_2} + \frac{V''_b + 10}{R_3} = 0$$

$$V''_b\left(\frac{1}{2} + \frac{1}{12} + \frac{1}{8} + \frac{1}{8}\right) + \frac{10}{8} + \frac{10}{8} = 0$$

$$V''_b\left(\frac{40}{48}\right) = -\frac{5}{2}$$

$$V''_b = -\frac{5}{2} \times \frac{48^{\,24\,3}}{40_{\,8}}$$

$$V''_b = -3\,V$$

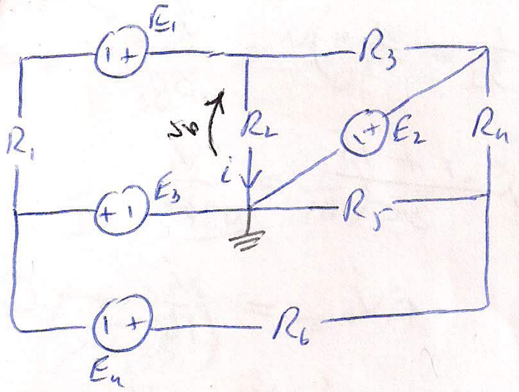
$V''_a = 0$

$V''_b = -3\,V$

$V''_c = 7\,V$

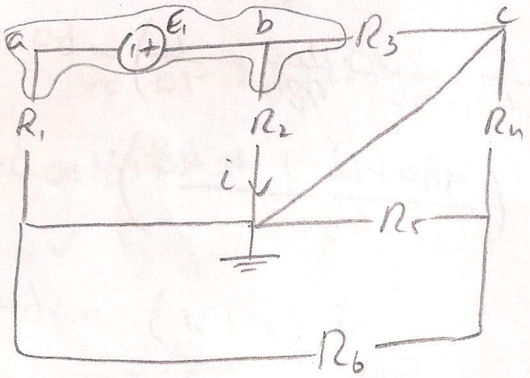
Hence

$$\left.\begin{aligned} V_a &= V'_a + V''_a = 12 + 0 = 12\,V \\ V_b &= V'_b + V''_b = 9 - 3 = 6\,V \\ V_c &= V'_c + V''_c = 9 + 7 = 16\,V \end{aligned}\right\} A_\Sigma$$



$i = ?$, by superposition

— E_1



$V'_c = 0$, $V'_b = V'_a + E_1$

$$\frac{V'_a}{R_1} + \frac{V'_b}{R_2} + \frac{V'_b}{R_3} = 0$$

$$\frac{V'_b - E_1}{R_1} + \frac{V'_b}{R_2} + \frac{V'_b}{R_3} = 0$$

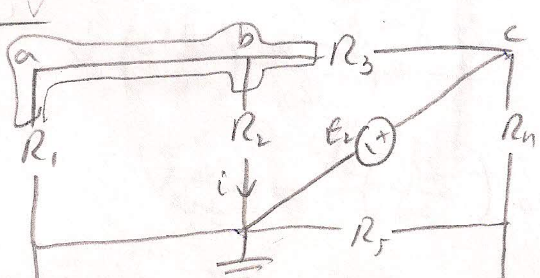
$$V'_b\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30}\right) = 1$$

$$V'_b\left(\frac{6+3+2}{60}\right) = 1$$

$$V'_b = \frac{60}{11} \Rightarrow V'_b = 5.45V$$

$$\boxed{V'_b = 5.45V}$$

— E_2



$V''_c = 20V$, $V''_a = V''_b$

$$\frac{V''_b}{R_1} + \frac{V''_b}{R_2} + \frac{V''_b - 20}{R_3} = 0$$

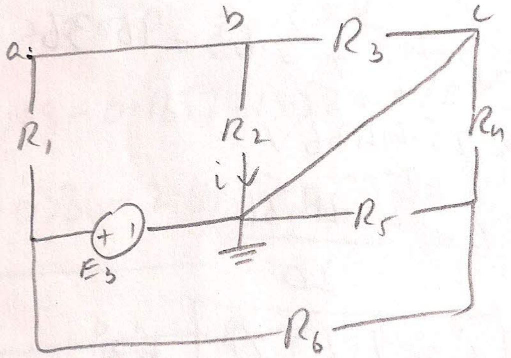
$$V''_b\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30}\right) = \frac{2}{3}$$

$$V''_b\left(\frac{11}{\cancel{60}\;20}\right) = \frac{2}{\cancel{3}}$$

$$V''_b = \frac{40}{11}$$

$$\boxed{V''_b = 3.63V}$$

— E_3



$V'''_c = 0$ $V'''_a = V'''_b$

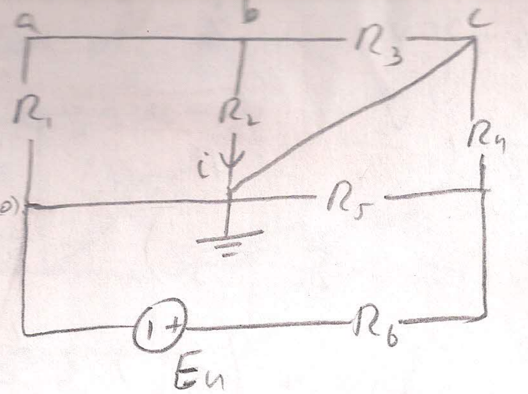
$$\frac{V'''_b - 30}{10} + \frac{V'''_b}{20} + \frac{V'''_b}{30} = 0$$

$$V'''_b\left(\frac{11}{60}\right) = 3$$

$$V'''_b = \frac{180}{11}$$

$$\boxed{V'''_b = 16.36V}$$

$-E_4$



No contribution of E_4 in voltage at point (b).

Hence

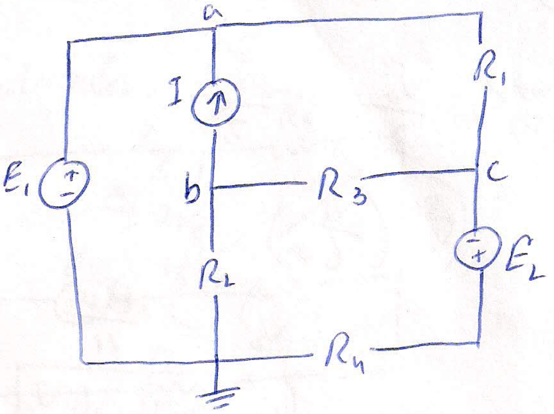
$V_b = 5.45 + 3.63 + 16.36$

$V_b = 25.4436\,V$

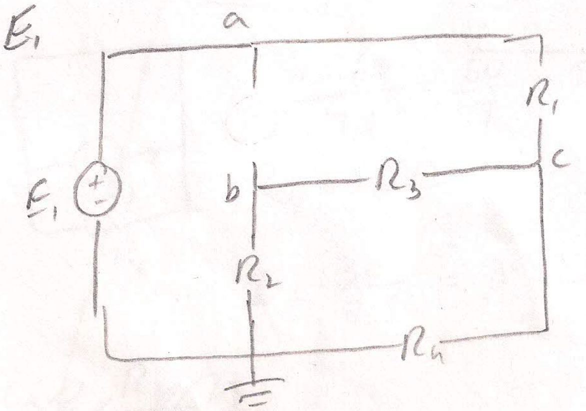
$$i = \frac{25.4436}{20}$$

$$\boxed{i = 1.27\,A}$$

2.16



$-E_1$



$V'_a = E_1 = 60V$

b) $$\frac{V'_b - V'_c}{R_3} + \frac{V'_b}{R_2} = 0$$

$$V'_b\left(\frac{1}{30} + \frac{1}{2}\right) = \frac{V'_c}{30}$$

$$V'_b\,\frac{1+15}{30} = \frac{V'_c}{30}$$

$$V'_b = \frac{V'_c}{16} \quad \text{— (1)}$$

c) $$\frac{V'_c - 60}{R_1} + \frac{V'_c - V'_b}{R_3} + \frac{V'_c}{R_4} = 0$$

$$V'_c\left(\frac{1}{1} + \frac{1}{30} - \frac{1}{480} + \frac{1}{10}\right) - 60 = 0$$

$$V'_c\left(\frac{480+16-1+48}{480}\right) = 60$$

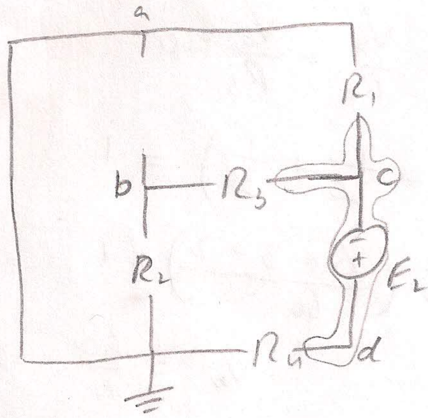
$$V'_c(1.131) = 60$$

$$V'_c = 53.0386\,V$$

$V'_b = 3.3149V$

$V'_a = 60V$

$-E_2$



$V''_a = 0$, $V''_d = V''_c + 30$

b) $$\frac{V''_b}{2} + \frac{V''_b - V''_c}{R_3} = 0$$

$$V''_b\left(\frac{16}{30}\right) = \frac{V''_c}{30}$$

$$V''_b = \frac{V''_c}{16} \quad \text{— (1)}$$

$$\frac{V_c''}{1} + \frac{V_c''-V_b''}{30} + \frac{V_c''+30}{R_4} = 0$$

$$V_c''\left(1+\frac{1}{30}-\frac{1}{480}+\frac{1}{10}\right) = -3$$

$$V_c''\left(\frac{543}{488}\right) = -3$$

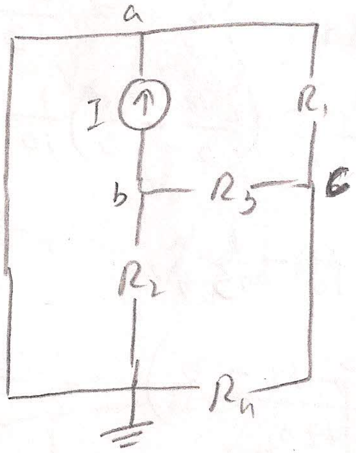
$$V_c'' = \frac{-3}{1.131}$$

$$V_c'' = -2.6525V$$

$$V_b'' = -0.1657V$$

$$V_a'' = 0V$$

— I



$$V_a''' = 0$$

b) $$7 + \frac{V_b'''-V_c'''}{30} + \frac{V_b'''}{2} = 0$$

$$V_b'''\left(\frac{1}{30}+\frac{1}{2}\right) = \frac{V_c'''}{30} - 7$$

$$V_b'''\left(\frac{16}{\cancel{30}}\right) = \frac{V_c'''-216}{\cancel{30}}$$

$$V_b''' = \frac{V_c'''-216}{16}$$

c) $$\frac{V_c'''}{1} + \frac{V_c'''-V_b'''}{30} + \frac{V_c'''}{10} = 0$$

$$V_c'''\left(1+\frac{1}{30}+\frac{1}{10}\right) - \frac{V_c'''}{16\cdot 30} + \frac{216}{480} = 0$$

$$V_c'''\left(1+\frac{1}{30}-\frac{1}{480}+\frac{1}{10}\right) = -\frac{216}{480}$$

$$V_c''(1.131) = -0.45$$

$$V_c''' = -0.3978$$

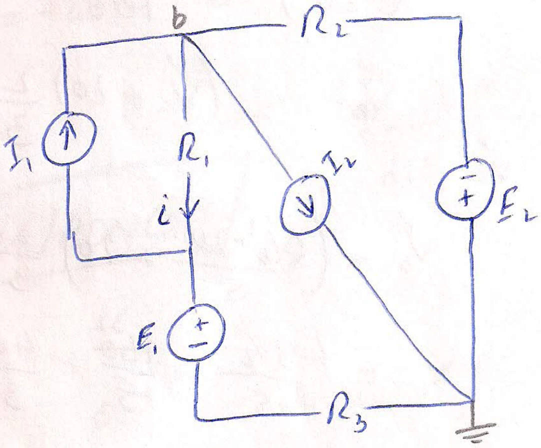
$$V_b''' = -13.52V$$

$$V_a''' = 0$$

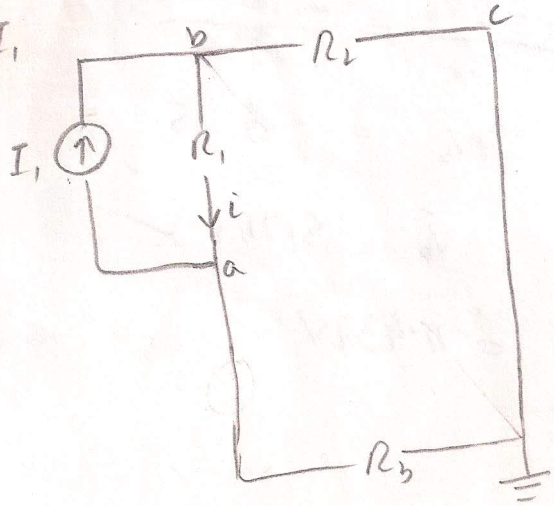
Hence

$$\left.\begin{aligned} V_a &= 60+0+0 = 60V \\ V_b &= 3.3149-0.1657-13.52 = -10V \\ V_c &= 53.0386-2.6525-0.3978 = 50V \end{aligned}\right\} A2$$

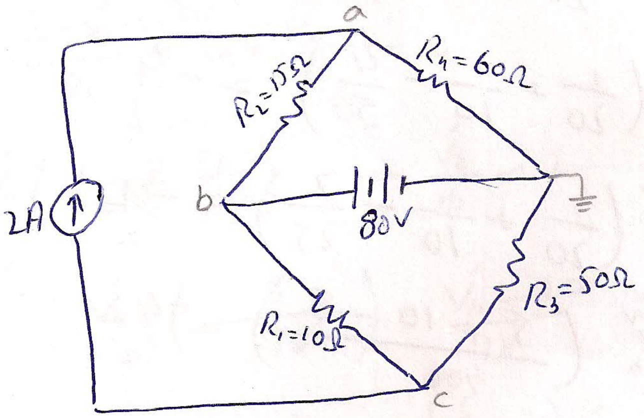
2.17



— I_1



2.18 $P(R_2) = ?$



$V_b = 80V$

a) $\frac{V_a - 80}{15} - 2 + \frac{V_a}{60} = 0$

$V_a\left(\frac{1}{15} + \frac{1}{60}\right) - \frac{\cancel{80}^{16}}{\cancel{15}_{3}} \cdot 1 - 2 = 0$

$V_a\left(\frac{4+1}{\cancel{60}_{20}}\right) = \frac{16+6}{3}$

$V_a = \frac{22 \times \cancel{3}\,\cancel{20}^{4}}{\cancel{5}}$

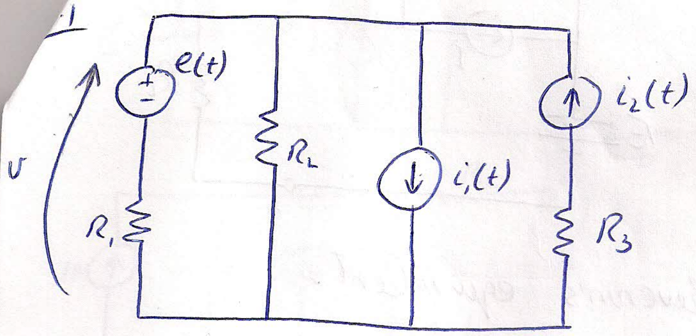
$V_a = 88V$

$P(R_2) = R_2\left(\frac{V_a - V_b}{R_2}\right)^2$

$= 15\left(\frac{8}{15}\right)^2 = \frac{64}{15}$

$\boxed{P(R_2) = 4.26\,W}$ Ans

CH#3



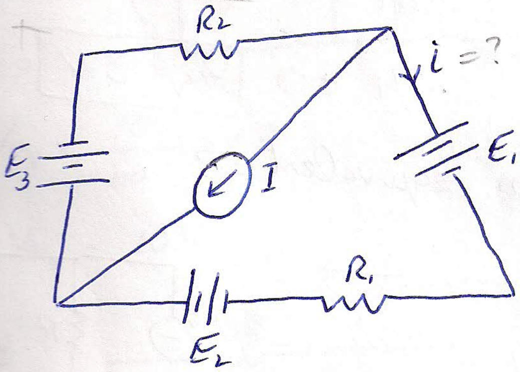
$$V = \frac{\frac{e(t)}{R_1} - i_1(t) + i_2(t)}{\frac{1}{R_1} + \frac{1}{R_2}}$$

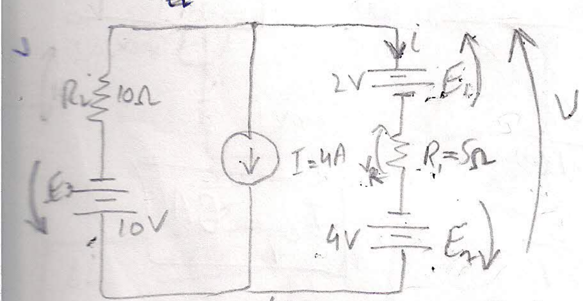
$$= \frac{\frac{10}{1/2} - 7 + 2}{\frac{1}{1/2} + \frac{1}{1/3}}$$

$$= \frac{20 - 7 + 2}{2 + 3} = \frac{15}{5}$$

$$\boxed{V = 3\ V}$$

3·2





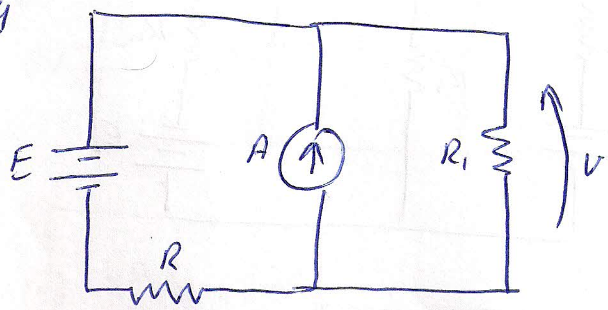
$$V = \frac{-4 - 1 - \frac{4-2}{5}}{\frac{1}{10} + \frac{1}{5}} = \frac{\frac{-20-5-2}{5}}{\frac{1+2}{10}} = \frac{-27}{3} \cdot 2 = -18V$$

KVL $-18 - 2 - V_R + 4 = 0 \Rightarrow V_R = -16$

$$i = \frac{V_R}{R_1} = \frac{-16}{5} \Rightarrow \boxed{i = -3.2A}$$

$\cancel{i = i_2 - I} \Rightarrow i = -3.2A$

3·4

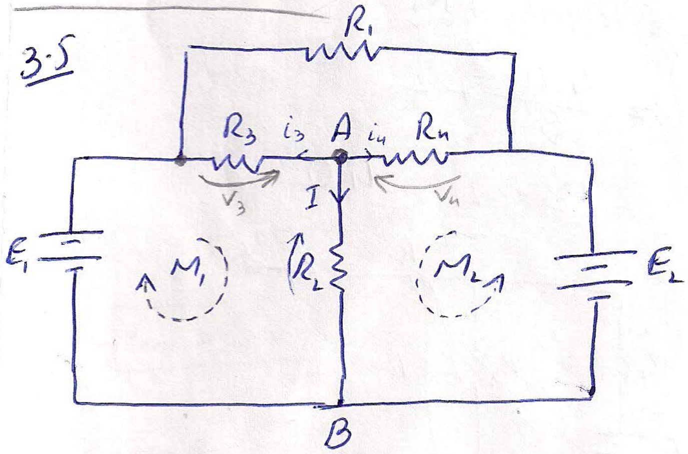


$$V = \frac{\frac{E}{R} + A}{\frac{1}{R} + \frac{1}{R_1}}$$

$$V = \frac{E + RA}{R\left(\frac{R_1 + R}{RR_1}\right)}$$

$$V = \frac{E}{\frac{R_1}{R_1}} \Rightarrow \boxed{V = E}$$

3·5



KVL M_1
$E_1 + i_3 R_3 - I R_2 = 0$ —①

KVL M_2
$E_2 + i_4 R_4 - I R_2 = 0$ —②

By ① − ②

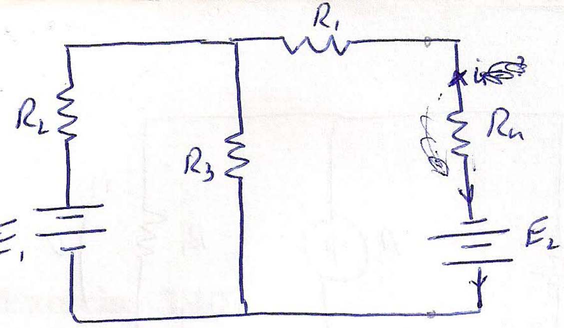
$E_1 - E_2 + i_3 R_3 - i_4 R_4 = 0$

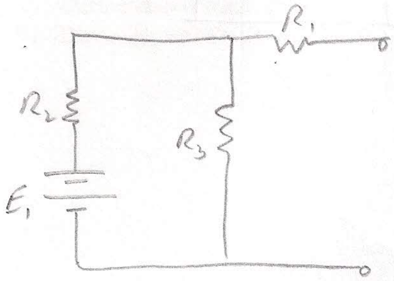
$4 - 8 + i_3 2 - i_4 4 = 0$

$i_3 - 2 i_4 = 2$

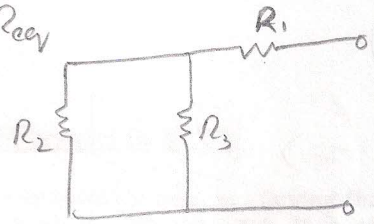
KCL

3.6





— R_{eq}

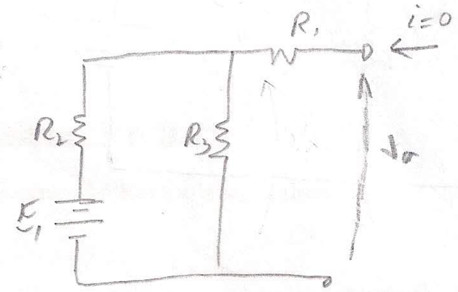


$$R_{eq} = R_2 \parallel R_3 + R_1$$

$$= \frac{4.4}{8} + 2$$

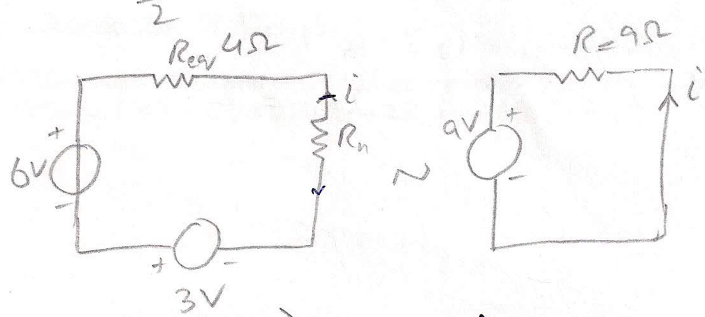
$$= 2 + 2 = 4\,\Omega$$

— V_a



$$V_a = \frac{\frac{E_1}{R_2}}{\frac{1}{R_2} + \frac{1}{R_3}} \Rightarrow V_a = \frac{\frac{12}{4}}{\frac{1}{4} + \frac{1}{4}}$$

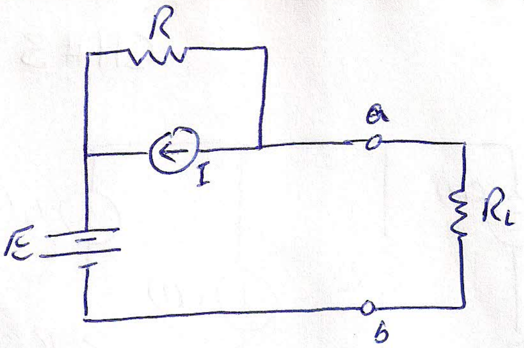
$$V_a = \frac{3}{\frac{1}{2}} \Rightarrow \boxed{V_a = 6\,V}$$



$$i = \left(V_{eq} / R_{eq} + R_n\right)$$

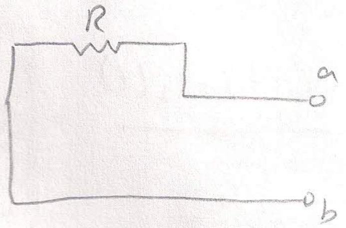
$$i = \frac{9}{9} \Rightarrow \boxed{i = 1\,A}$$

3.7



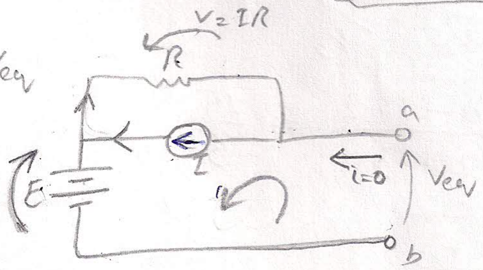
i) Thevenin's equivalent = ?

— R_{eq}



$$R_{eq} = R \Rightarrow \boxed{R_{eq} = 1\,\Omega}$$

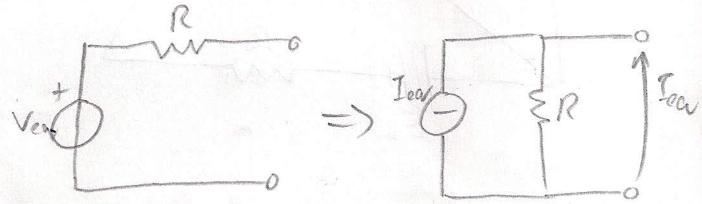
— V_{eq}



$$V_{eq} = E - IR$$

$$= 12 - 4 \Rightarrow \boxed{V_{eq} = 8\,V}$$

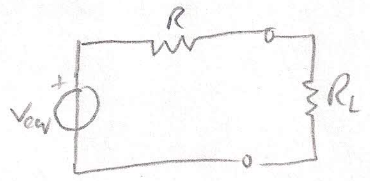
ii) Norton equivalent = ?



$$I_{eq} = \frac{V_{eq}}{R}$$

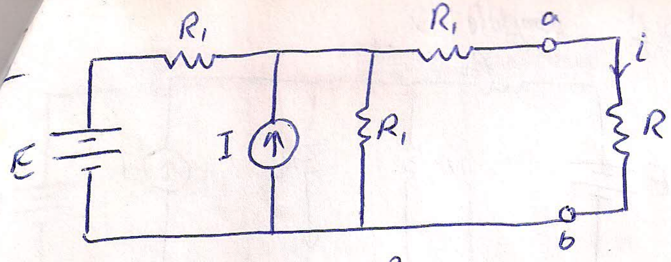
$$= \frac{8}{1} \Rightarrow \boxed{I_{eq} = 8A}$$

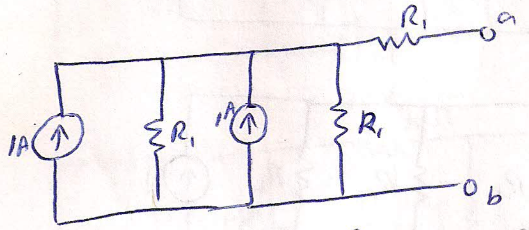
c)

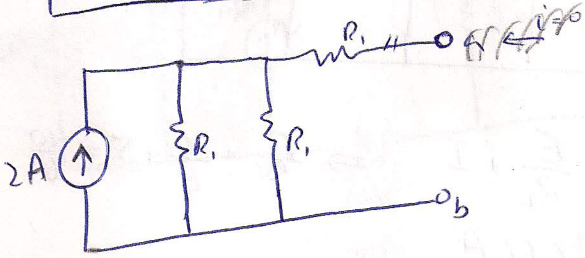


$$P = R_L \left(\frac{V_{eq}}{R + R_L}\right)^2 = \left(\frac{8}{1 + R_L}\right)^2 R_L$$

$$P = 64 \frac{R_L}{(1 + R_L)^2}$$



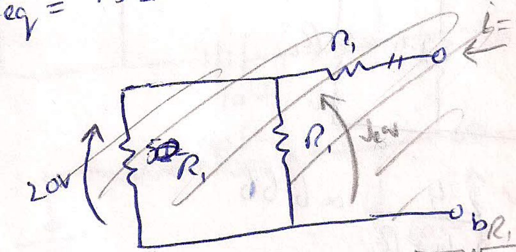


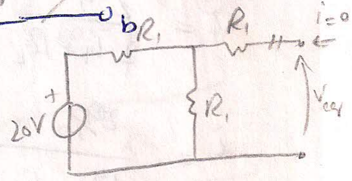


$$-R_{eq}\ \frac{10 \cdot 10}{20} + 10$$

$$R_{eq} = 15\,\Omega$$

$-V_{eq}$

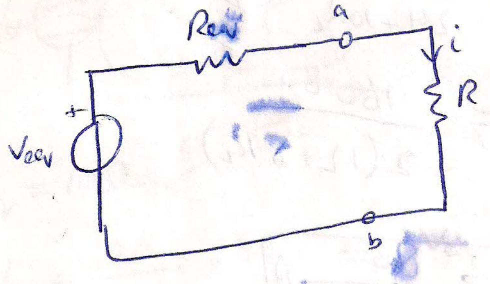




$$V_{eq} = \frac{10}{\cancel{20}} \times \cancel{20}$$

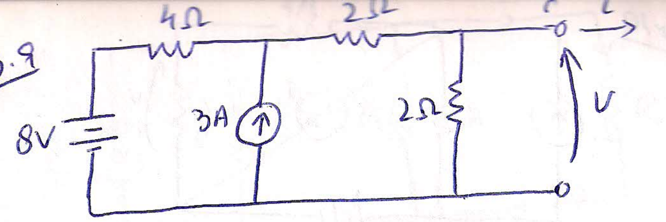
$$V_{eq} = 10\,V$$

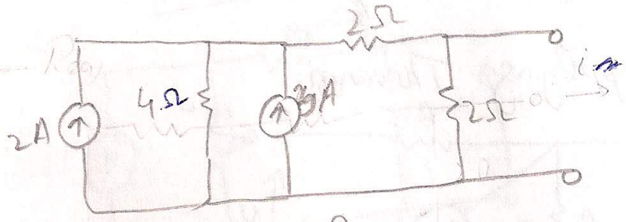
Now

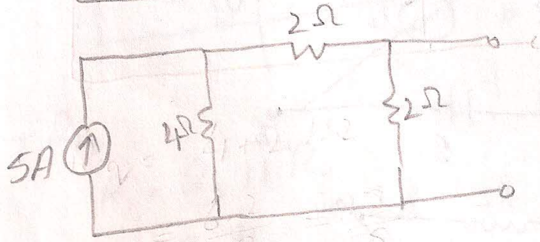


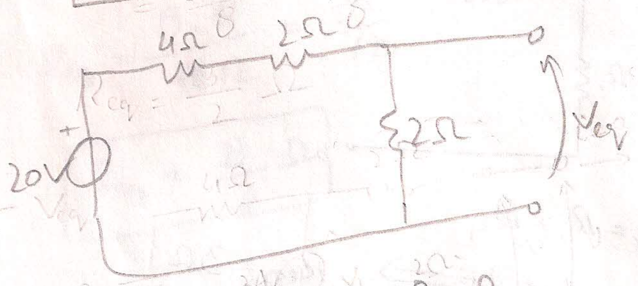
$$\boxed{i = \frac{10}{15 + R}\ A}$$

3.9









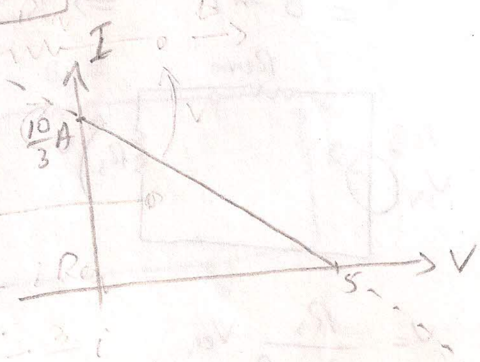
$$R_{eq} = \frac{6 \cdot 2}{8} = \frac{12}{8} = \frac{3}{2}\,\Omega$$

$$V_{eq} = \frac{2}{8_4} \times 20^{5}$$

$$\boxed{V_{eq} = 5\,V}$$

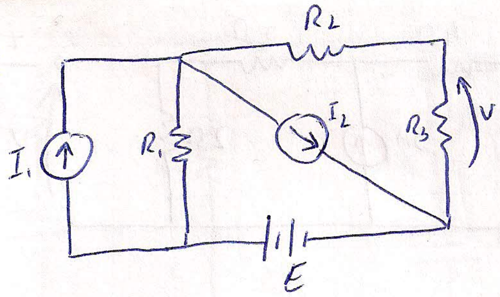
$$I_{eq} = \frac{V_{eq}}{R_{eq}} = \frac{5}{3/2}$$

$$\boxed{I_{eq} = \frac{10}{3}\ A}$$

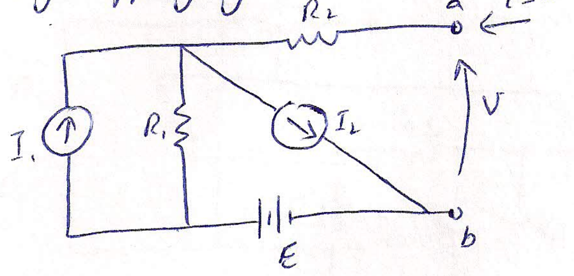


$V =$

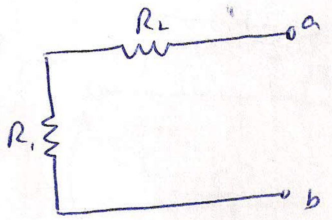
3.10



By applying Thevenin

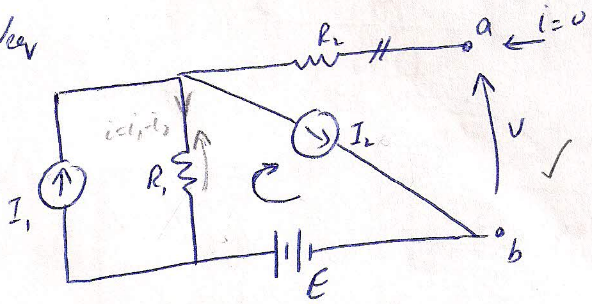


Req-

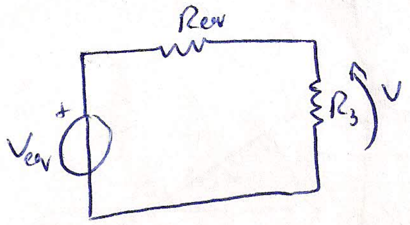


$$R_{eq} = R_1 + R_2$$
$$= 1 + 2 \Rightarrow \underline{R_{eq} = 3\Omega}$$

- Veq

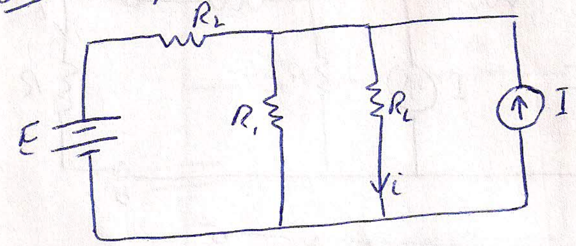


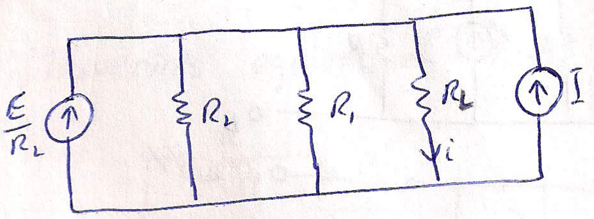
$$V_{eq} = E + R_1(I_1 - I_2)$$
$$= 6 + 1(4-5)$$
$$= 6 - 1 \Rightarrow \underline{V_{eq} = 5\,V}$$



$$V = \frac{R_3}{R_{eq}+R_3} V_{eq}$$
$$= \frac{3}{3+3} \times 5$$
$$V = \frac{15}{6} \Rightarrow \boxed{V = 2.5\,V}$$

3.11 compute i

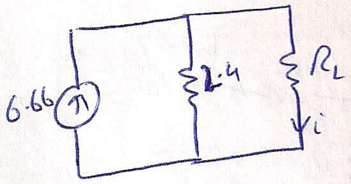




$$I_{eq} = \frac{E}{R_2} + I \Rightarrow I_{eq} = \frac{10}{6} + 5 = \frac{40}{6}$$

$$I_{eq} = 6.66\,A$$

$$R_1 \parallel R_2 = \frac{24}{10} = 2.4\Omega$$



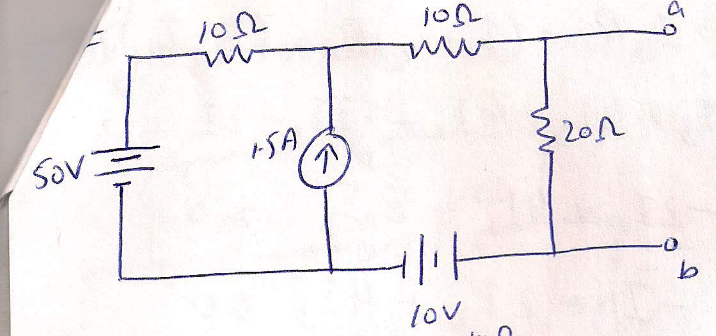
$$i = \frac{2.4}{2.4 + R_L} \times 6.66$$

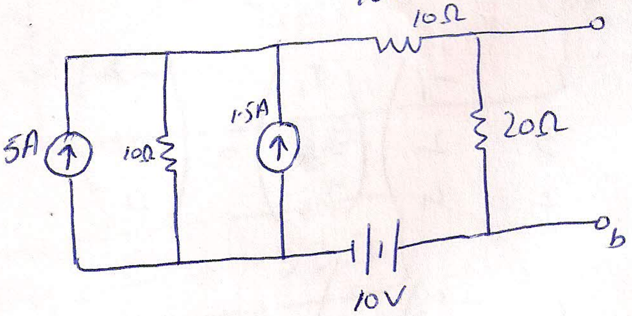
$$i = \frac{24/10}{\frac{24}{10} + R_L} \times \frac{40}{6}$$

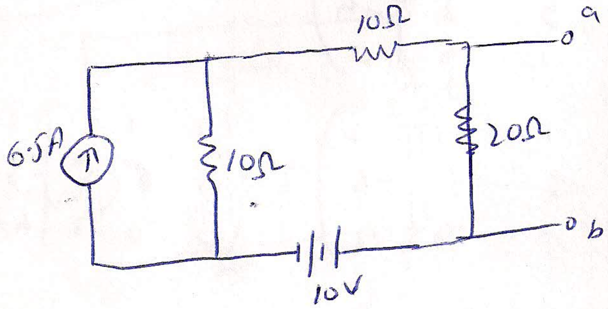
$$= \frac{24}{24 + 10R_L} \times \frac{40}{6}$$

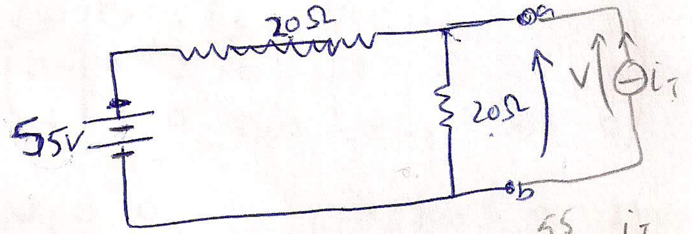
$$= \frac{\cancel{160}\ 80}{\cancel{2}(12 + 5R_L)}$$

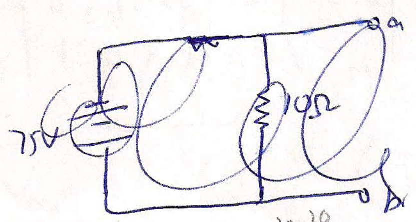
$$\boxed{i = \frac{80}{12 + 5R_L}\,A}$$











$V = \dfrac{\frac{55}{20} + i_T}{\frac{1}{10}}$

$V = \dfrac{55}{2} + 10 i_T$

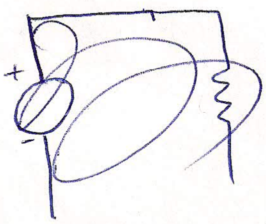
$V_{eq} = 27.5\,V$

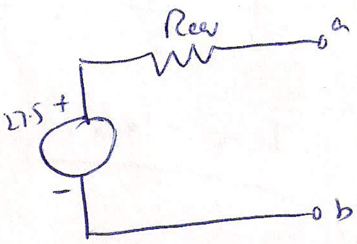
$R_{eq} = 10\,\Omega$

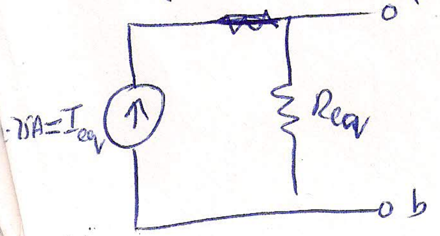
$\dfrac{20 \cdot 20}{20+20}$

$R_{eq} = 10\,\Omega$

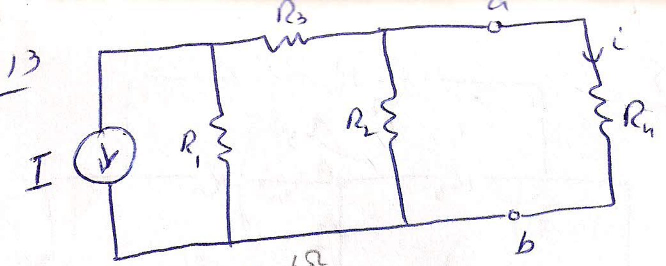
$V_{eq} = \dfrac{20}{20+20} \times 55 \Rightarrow V_{eq} = 27.5\,V$







3.13



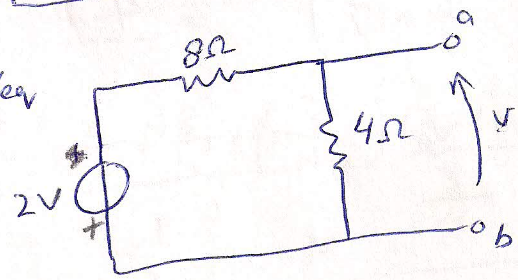
— R_{eq}

$R_1 + R_3 \parallel R_2$

$R_{eq} = \dfrac{8 \cdot 4}{12} = \dfrac{32}{12} = \dfrac{8}{3}$

$\boxed{R_{eq} = \frac{8}{3}\,\Omega}$

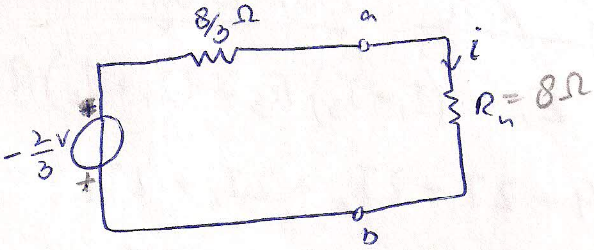
— V_{eq}



$V_{eq} = -\dfrac{4}{12} \times 2$

$= -\dfrac{8}{12}$

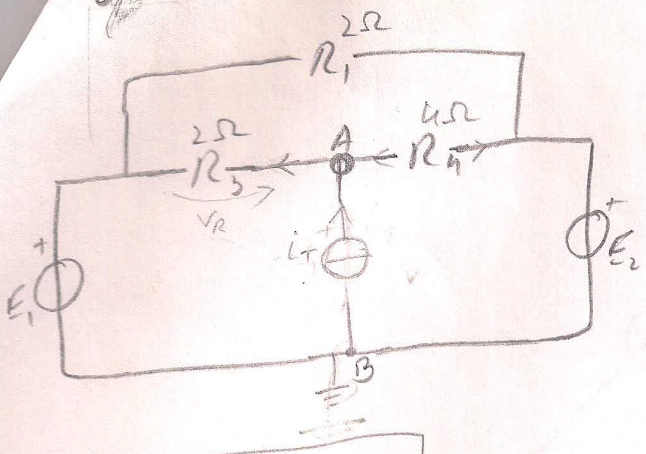
$\boxed{V_{eq} = -\frac{2}{3}\,V}$



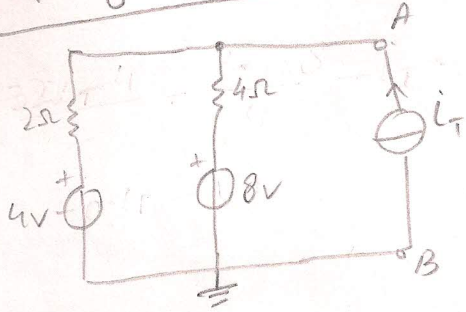
$i = \dfrac{-2/3}{8/3 + 8} = -\dfrac{2}{32}$

$\boxed{i = \frac{-1}{16}\,A}$

...ly Thevenin



$$\boxed{R_{eq} = \frac{8}{6} = 1.33\,\Omega}$$



KCL

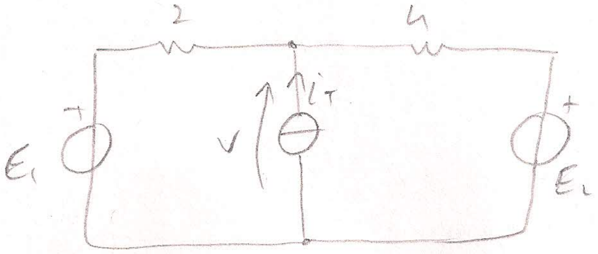
$$\frac{4}{2} + \frac{8}{2} - i_T = 0$$

$$i_T = 4A$$

$$V_{eq} = i_T R_{eq}$$

$$= 4(1.33)$$

$$\boxed{V_{eq} = 5.3V}$$ A2



$$\frac{V - E_1}{2} + \frac{V - E_2}{4} - i_T = 0$$

$$V\left(\frac{1}{2} + \frac{1}{4}\right) - 2 - 2 - i_T = 0$$

$$V\left(\frac{3}{4}\right) = 4 + i_T$$

$$V = 5.33 + 1.33\, i_T$$

$$\boxed{\begin{aligned} V_{eq} &= 5.33\,V \\ R_{eq} &= 1.33\,\Omega \end{aligned}}$$

OR

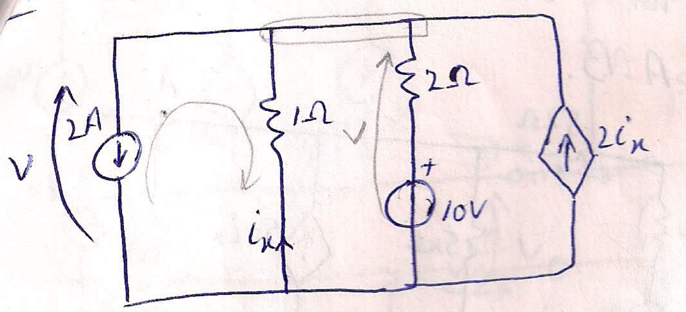
$$V = \frac{\frac{4}{2} + \frac{8}{4} + i_T}{\frac{1}{2} + \frac{1}{4}} = \frac{4 + i_T}{\frac{3}{4}}$$

$$V = \frac{16}{3} + \frac{4}{3} i_T$$

$$\boxed{\begin{aligned} R_{eq} &= \frac{4}{3}\,\Omega \\ V_{eq} &= \frac{16}{3}\,V \end{aligned}}$$

Circuits With Dependent Sources

4.1 $V = ?$



KCL $2 - i_x + \frac{V-10}{2} - 2i_x = 0$

$\frac{V}{2} - 5 + 2 - 3i_x = 0$

$\frac{V}{2} = 3(i_x + 1)$

KVL

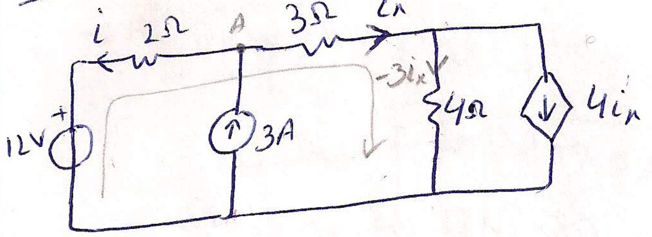
$V + i_x = 0 \Rightarrow i_x = -V$

$\frac{V}{2} = -3V + 3$

$\frac{V}{2} + 3V = 3 \Rightarrow 7V = 6$

$V = \frac{6}{7} \Rightarrow \boxed{V = 0.8571V}$

4.2 compute (i)



KCL @ A

$i + i_x = 3 \Rightarrow i_x = 3 - i$

KVL $12 + 2i - 3i_x - 4(-3i_x) = 0$

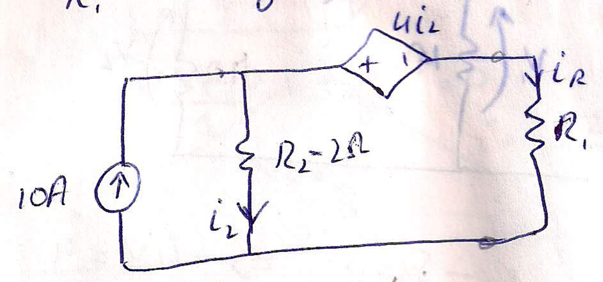
$12 + 2i - 3i_x + 12i_x = 0$

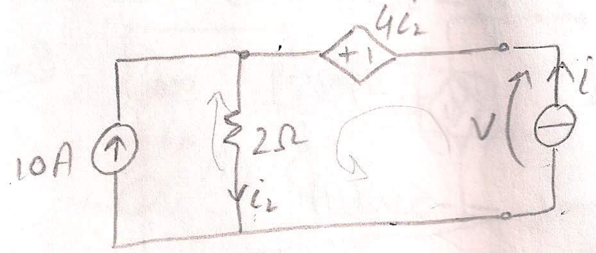
$12 + 2i + 9i_x = 0$

$12 + 2i + 27 - 9i = 0$

$7i = 39 \Rightarrow \boxed{i = 5.57A}$

4.3 Evaluate i_R as a function of R_1 using Thevenin's theorem.





KCL $-10 + i_2 - i_T = 0$

$i_2 = i_T + 10$

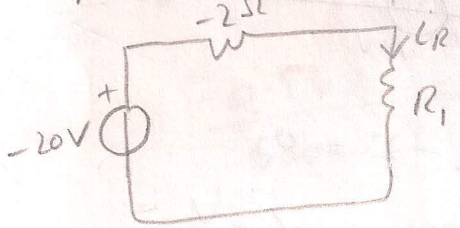
KVL $V + 4i_2 - 2i_2 = 0$

$V + 2(i_T + 10) = 0$

$V = -20 - 2i_T$

$V_{eq} = -20V$

$R_{eq} = -2Ω$

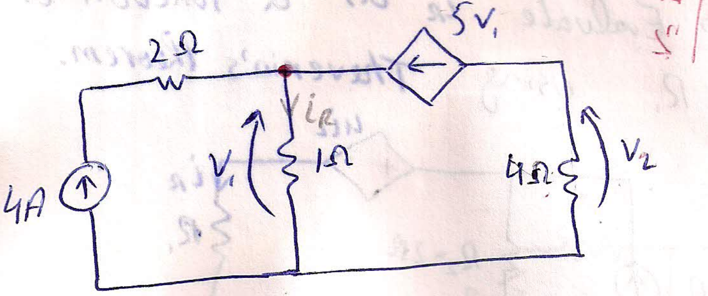


$i_R = \frac{-20}{-2 + R_1}$

$\boxed{i_R = \frac{20}{2 - R_1} A}$

4.4 Compute V_2

$V = IR$



KCL $-4 + \frac{V_1}{1} - 5V_1 = 0 \Rightarrow -4V_1 = 4$

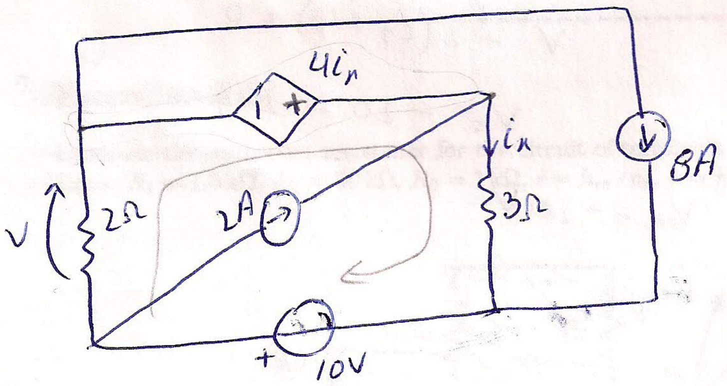
$$\boxed{V_1 = -1\,V}$$

KVL $V_2 = -5V_1(4)$

$$V_2 = -5(-1)(4)$$

$$\boxed{V_2 = 20\,V}$$

4.5 Compute V



KCL
$$8 + \frac{V}{2} + i_x - 2 = 0$$

$$i_x = -6 - \frac{V}{2}$$

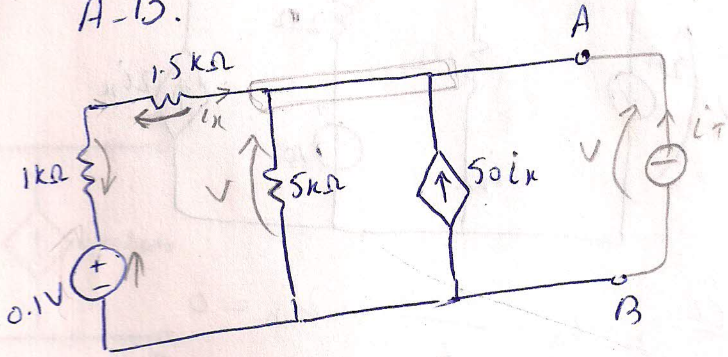
KVL $V + 4i_x - 3i_x + 10 = 0$

$$V - 6 - \frac{V}{2} + 10 = 0$$

$$\frac{V}{2} = -4$$

$$\boxed{V = -8\,V}$$ Ans

4.6 Build thevenin equivalent for the circuit of terminals A-B.



KCL $-i_T - 50i_x + \frac{V}{5\times10^3} - i_x = 0$

$$51i_x = \frac{V}{5\times10^3} - i_T$$

$$i_x = \left(\frac{V - 5k\,i_T}{5k}\right)\frac{1}{51}$$

KVL $1.5i_x + i_x - 0.1 + V = 0$

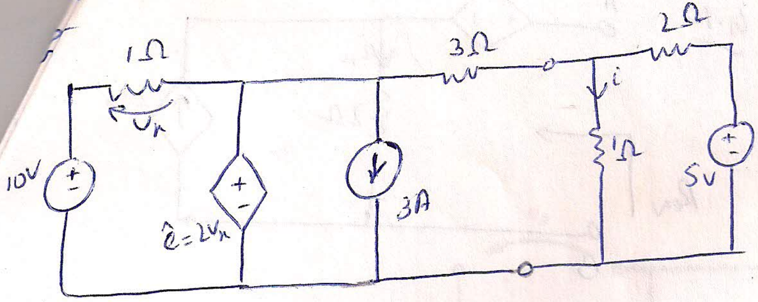
$$\frac{2.5k}{51}\left(\frac{V - 5k\,i_T}{5k}\right) + V = 0.1$$

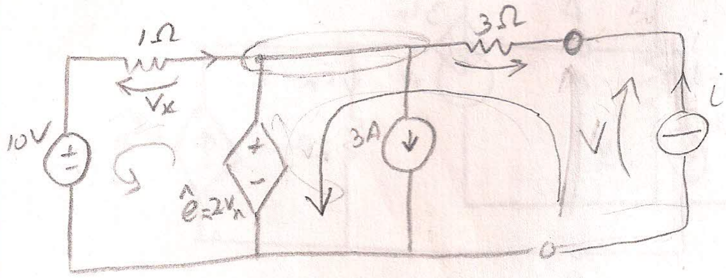
$$0.0098036V - 49.019i_T + V = 0.1$$

$$1.0098036V = 0.01 + 49.019i_T$$

$$V = 0.0099 + 48.54i_T$$

$$\boxed{\begin{array}{l} V_{eq} = 0.0099\,V \\ R_{eq} = 48.54\,\Omega \end{array}}$$





KVL $V_x - 10 + 2V_x = 0$

$3V_x = 10 \Rightarrow \boxed{V_x = \frac{10}{3}}$

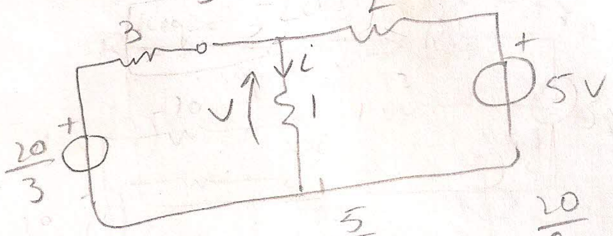
$(-2V_x)$

$V - 3i_T - \frac{20}{3} = 0$

$V = 3i_T + \frac{20}{3}$

$R_{eq} = 3\Omega$

$V_{eq} = \frac{20}{3} V$

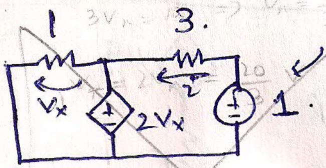


$$V = \frac{\frac{20}{3}/3 + \frac{5}{2}}{\frac{1}{3} + 1 + \frac{1}{2}} = \frac{\frac{20}{9} + \frac{5}{2}}{\frac{2+6+3}{6}}$$

$$V = \frac{\frac{40+45}{18}}{\frac{11}{6}} = \frac{85}{33} V$$

$i = \frac{V}{1} \Rightarrow \boxed{i = 2.57A}$

for Req



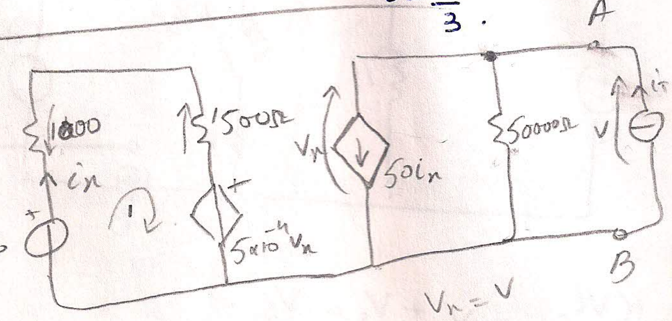
$$\frac{2V_x - 0}{1} + \frac{2V_x - 1}{3} = 0$$

$V_x = \frac{1}{8}$

$i = \frac{1 - 2V_x}{3} = \frac{1 - \frac{1}{4}}{3}$

$i = \frac{1}{3}$

4.8



$V_x = V$

KVL(1) $V_o - i_x(2500) - 0.0005 V_x = 0$

$i_x = \frac{V_o}{2500} - \frac{0.0005}{2500} V_x$

KCL $50 i_x + \frac{V}{50000} = i_T$

$50\left(\frac{V_o - 0.0005V}{2500}\right) + \frac{V}{50000} = i_T$

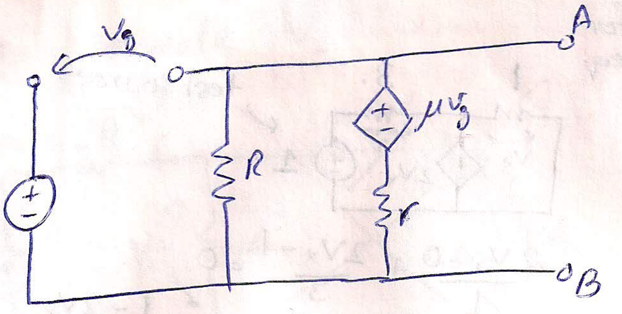
$\frac{V_o}{50} - 1\times10^{-5} V + \frac{V}{50000} = i_T$

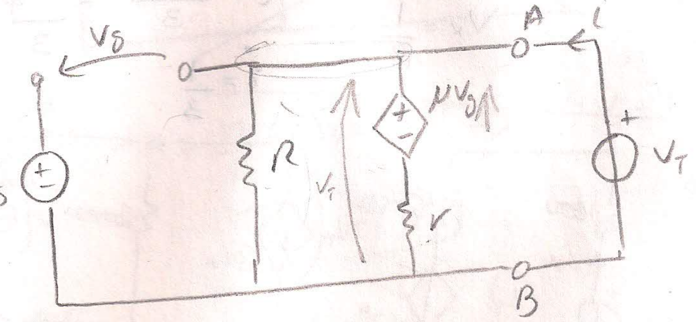
$\left(\frac{-0.5 + 1}{50000}\right) V = i_T - \frac{V_o}{50}$

$V = 100000 i_T - \frac{V_o}{5\times10^{-4}}$

$\boxed{R_{eq} = 100 k\Omega}$

4.9





KVL $\quad V_T + V_g = V_s$

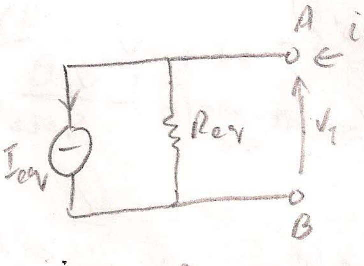
$$V_g = V_s - V_T$$

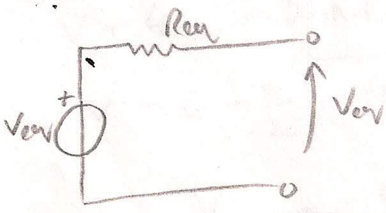
KCL $\quad -i + \frac{V_T}{R} + \frac{V_T - \mu V_g}{r}$

$$i = \frac{V_T}{R} + \frac{V_T}{r} - \frac{\mu(V_s - V_T)}{r}$$

$$i = \underbrace{V_T\left(\frac{1}{R} + \frac{1+\mu}{r}\right)}_{G_{eq}} - \underbrace{\frac{\mu V_s}{r}}_{I_{eq}}$$

$$G_{eq} = \frac{r + R(1+\mu)}{rR}, \quad I_{eq} = -\frac{\mu V_s}{r}$$

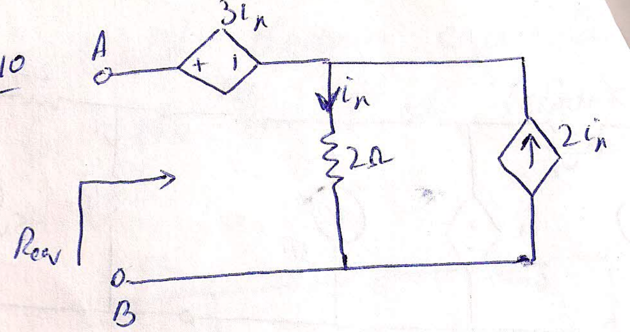


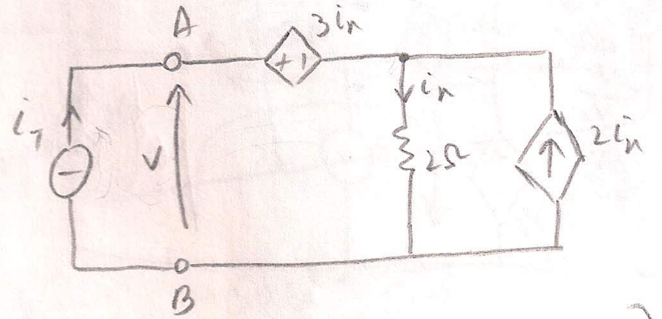


$$\boxed{R_{eq} = \frac{rR}{r + R(1+\mu)}} \quad \boxed{V_{eq} = \frac{\mu V_s R}{r + R(1+\mu)}}$$

$$V_{eq} = -I_{eq} R_{eq} = \frac{\mu V_s}{r} \cdot \frac{rR}{r + R(1+\mu)}$$

4.10





$V_{eq} = 0$ (No independent source)

KCL $\quad i_T = i_x - 2i_x$

$$i_T = -i_x \Rightarrow i_x = -i_T$$

KVL $\quad V = 3i_x + 2i_x$

$$V = 5i_x$$

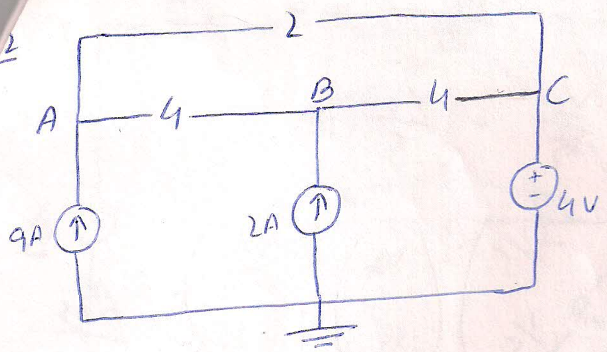
$$V = -5i_T$$

$$R_{eq} = \frac{V}{i_T}$$

$$= \frac{-5i_T}{i_T}$$

$$\boxed{R_{eq} = -5\,\Omega}$$

$$V = R_{eq} + i_T V_{eq}$$



$V_C = 4V$

A) $\frac{V_A - V_B}{4} - 9 + \frac{V_A - 4}{2} = 0$

$V_A\left(\frac{1}{4} + \frac{1}{2}\right) = 11 + \frac{V_B}{4}$

$V_A = \frac{44}{3} + \frac{V_B}{3}$

B) $\frac{V_B - 4}{4} + \frac{V_B - V_A}{4} - 2 = 0$

$V_B\left(\frac{1}{4} + \frac{1}{4}\right) - \frac{11}{3} - \frac{V_B}{12} - 3 = 0$

$V_B\left(\frac{1}{4} + \frac{1}{4} - \frac{1}{12}\right) = \frac{20}{3}$

$V_B\left(\frac{3+3-1}{12}\right) = \frac{20}{3}$

$V_B = \frac{80}{5}$

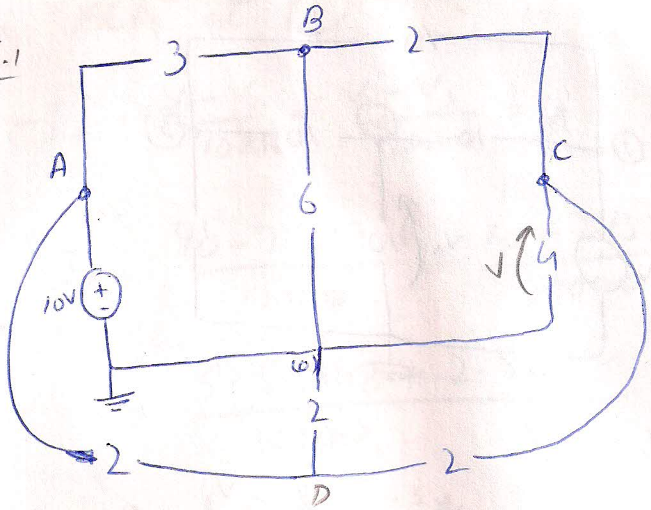
$\boxed{V_B = 16V}$

$V_A = \frac{44}{3} + \frac{16}{3}$

$V_A = \frac{60}{3}$

$\boxed{V_A = 20V}$

5.1



$V_A = 10V$

B) $\frac{V_B - 10}{3} + \frac{V_B}{6} + \frac{V_B - V_C}{2} = 0$

$V_B\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\right) = \frac{10}{3} + \frac{V_C}{2}$

$V_B\left(\frac{2+1+3}{6}\right) = \frac{10}{3} + \frac{V_C}{2}$

$V_B = \frac{10}{3} + \frac{V_C}{2}$ ✓

C) $\frac{V_C}{4} + \frac{V_C - V_D}{2} + \frac{V_C - V_B}{2} = 0$

$V_C\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{2}\right) - \frac{10}{6} - \frac{V_C}{4} = 0$

$V_C = \frac{5}{3} + \frac{V_D}{2}$ ✓

D) $\frac{V_D - 10}{2} + \frac{V_D}{2} + \frac{V_D - V_C}{2} = 0$

$V_D\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right) - \frac{5}{6} - \frac{V_D}{4} = 5$

$V_D\left(\frac{2+2+2-1}{4}\right) = 5 + \frac{5}{6}$

$V_D = \left(5 + \frac{5}{6}\right)\frac{4}{5}$

$V_D = 4 + \frac{2}{3}$

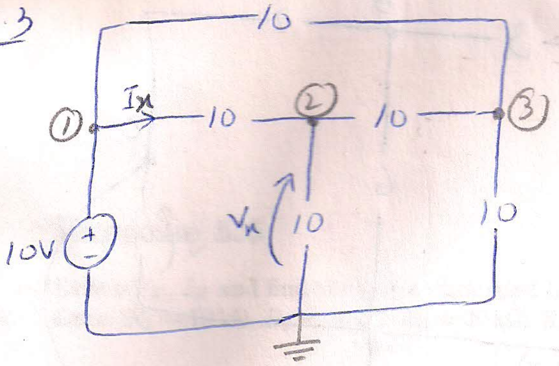
$V_D = \frac{14}{3}V$ ✓

$V_C = \frac{5}{3} + \frac{14}{6} = \frac{10+14}{6} = \frac{24}{6}$

$V_C = 4V$

$V = V_C - 0 \Rightarrow \boxed{V = 4V}$

5.3



$V_1 = 10V$

2) $\dfrac{V_2 - 10}{10} + \dfrac{V_2}{10} + \dfrac{V_2 - V_3}{10} = 0$

$V_2\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right) - \frac{V_3}{10} = 1$

$\frac{3}{10} V_2 - \frac{V_3}{10} = 1$

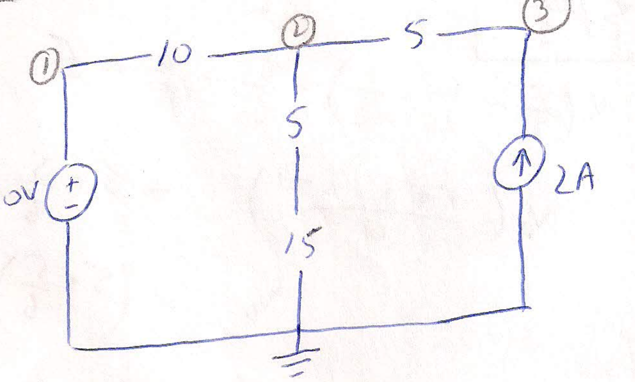
3) $\dfrac{V_3 - 10}{10} + \dfrac{V_3 - V_2}{10} + \dfrac{V_3}{10} = 0$

$V_3\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right) - \frac{V_2}{10} = 1$

$-\frac{V_2}{10} + \frac{3}{10} V_3 = 1$

$$\begin{pmatrix} 3/10 & -1/10 \\ -1/10 & 3/10 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

5.4



$V_1 = 10V$

2) $\dfrac{V_2 - 10}{10} + \dfrac{V_2}{20} + \dfrac{V_2 - V_3}{5} = 0$

$V_2\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{5}\right) - \frac{V_3}{5} = 1$

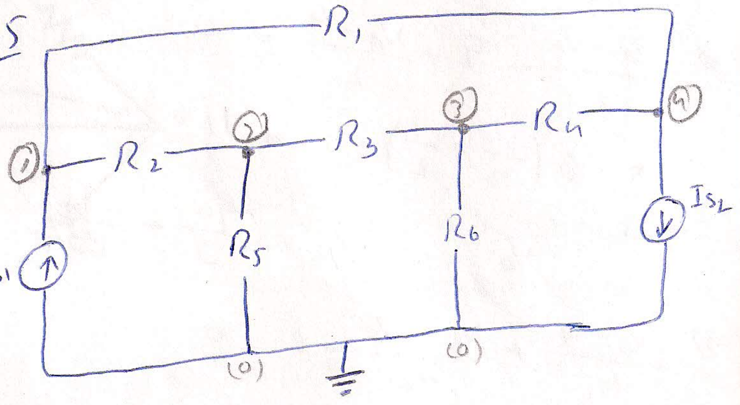
$V_2 \frac{7}{20} - \frac{V_3}{5} = 1$

3) $-2 + \dfrac{V_3 - V_2}{5} = 0$

$-\frac{V_2}{5} + \frac{V_3}{5} = 2$

$$\begin{pmatrix} 7/20 & -1/5 \\ -1/5 & 1/5 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ Ans

5.5



1) $\dfrac{V_1 - V_4}{1000} + \dfrac{V_1 - V_2}{1000} - 100 = 0$

$\frac{1}{1000}(2V_1 - V_2 - V_4) = 100$

2) $\dfrac{V_2}{1000} + \dfrac{V_2 - V_1}{1000} + \dfrac{V_2 - V_3}{1000} = 0$

$\frac{1}{1000}(-V_1 + 3V_2 - V_3) = 0$

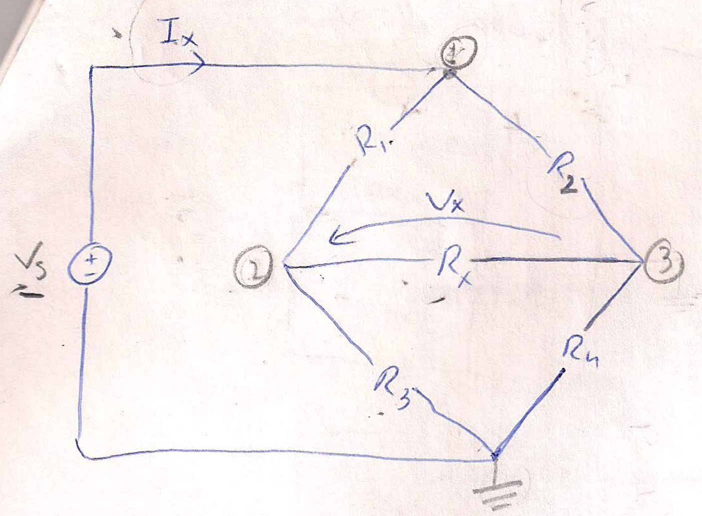
3) $\dfrac{V_3 - V_2}{1000} + \dfrac{V_3 - V_4}{1000} + \dfrac{V_3}{1000} = 0$

$\frac{1}{1000}(-V_2 + 3V_3 - V_4) = 0$

4) $\dfrac{V_4 - V_1}{1} + \dfrac{V_4 - V_3}{1} + 100 = 0$

$\frac{1}{1000}(-V_1 - V_3 + 2V_4) = -100$

$$\frac{1}{1000}\begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \\ -100 \end{pmatrix} mA$$



$V_1 = 10V$

2) $\frac{V_2 - 10}{10000} + \frac{V_2 - V_3}{3000} + \frac{V_2}{30000} = 0$

$\cancel{\frac{1}{1000}} V_2 \left(\frac{1}{10} + \frac{1}{3} + \frac{1}{30}\right) = \cancel{\frac{1}{1000}}\left(\frac{V_3}{3} + 1\right)$

$V_2 \left(\frac{3 + 10 + 1}{\cancel{30}\ 10}\right) = \frac{V_3 + 3}{3}$

$V_2 = (V_3 + 3)\frac{\cancel{10}\ 5}{\cancel{14}\ 7}$

$V_2 = \frac{5}{7} V_3 + \frac{15}{7}$

3) $\frac{V_3 - 10}{20} + \frac{V_3 - V_2}{3} + \frac{V_3}{40} = 0$

$V_3\left(\frac{1}{20} + \frac{1}{3} + \frac{1}{40}\right) - \frac{5}{21} V_3 - \frac{15}{21} - \frac{1}{2} = 0$

$V_3\left(\frac{1}{20} + \frac{1}{3} + \frac{1}{40} - \frac{5}{21}\right) = \frac{5}{7} - \frac{1}{2} = 0$

$V_3\left(\frac{42 + 280 + 21 - 200}{\cancel{840}\ 60}\right) = \frac{+10 + 7}{\cancel{14}}$

$V_3\left(\frac{143}{60}\right) = 17$

$V_3 = 7.13286V$

$V_2 = 7.23776$

$V_x = V_2 - V_3$

$\boxed{V_x = 0.105\,V}$

By KCL at 1

$-I_x + \frac{V_1 - V_2}{10k\Omega} + \frac{V_1 - V_3}{20000} = 0$

$I_x = \frac{10 - 7.23776}{10000} + \frac{10 - 7.13286}{20000}$

$= \frac{5.52448 + 2.86714}{20000}$

$I_x = 0.419 \times 10^{-3} A$

$\boxed{I_x = 0.419\,mA}$

$P = VI$

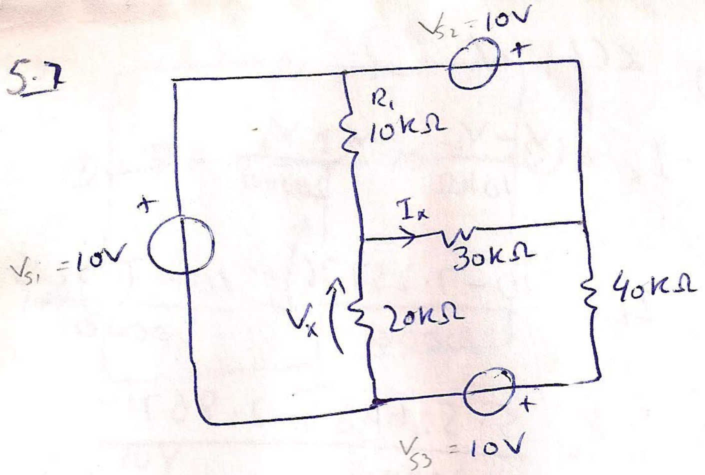
$P = \frac{V^2}{R_2}$

$P = \frac{(V_1 - V_3)^2}{20000}$

$P = \frac{8.22049}{20} \times 10^{-3}$

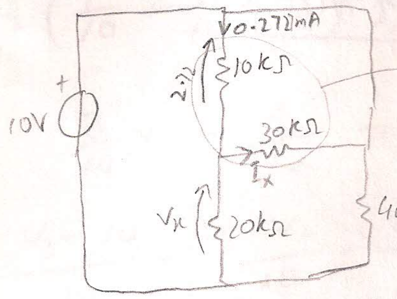
$\boxed{P = 0.411\,mW}$

5.7

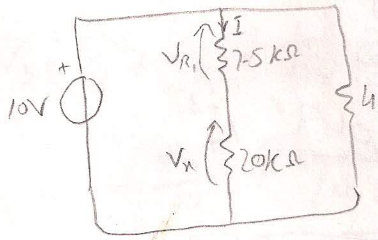


By superposition

– Contribution of V_{s1}



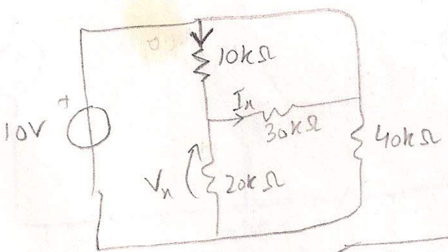
$\frac{30 \cdot 10}{40} = 7.5k\Omega$



$V_x = \frac{20}{7.5+20} \times 10V \Rightarrow \boxed{V_x' = 7.27V}$

$V(7.5) = \frac{7.5}{27.5} \times 10 \Rightarrow V(7.5) = 2.7272V$

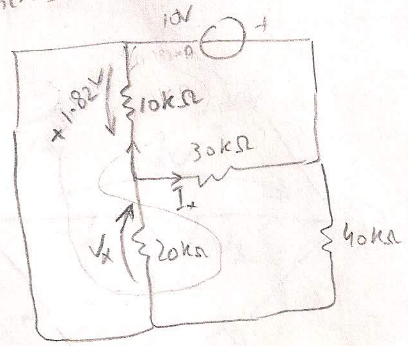
$I(7.5) = \frac{2.72}{7.5} \Rightarrow I(7.5) = 0.36mA$

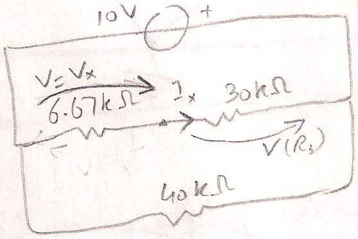


$I_x = -\frac{10}{10+30} \times 0.36m \Rightarrow \boxed{I_x' = -0.09mA}$

– Contribution of V_{s2}

$\frac{200}{30}$





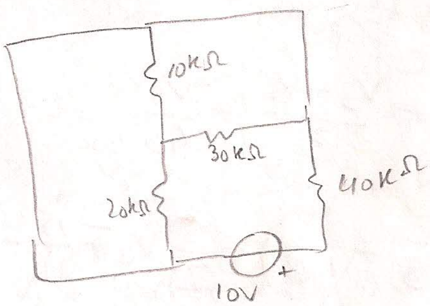
$V = V_x = \frac{6.67}{30+6.6} \times 10 \Rightarrow \boxed{V_x' = 1.82V}$

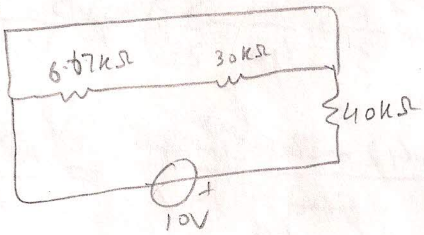
$V(R_3) = \frac{30}{36.67} \times 10 \Rightarrow V(R_3) = 8.18V$

$I_x = -\frac{V(R_3)}{R_3}$

$\boxed{I_x = -0.2727mA}$

– Contribution of V_{s3}





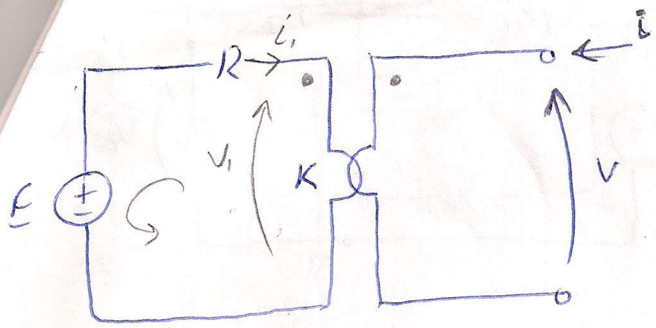
$I_x = V_x = 0$

$V_x = 7.27 + 1.82 = 9.09V$

$I_x = -0.09 \times 10^{-3} + -0.2727 \times 10^{-3} = -0.36mA$

$P(R_1) = R_1 I^2 = 10000(0.272 \mp 0.182)^2$

$\boxed{P(R) = 82.2\mu WA}$



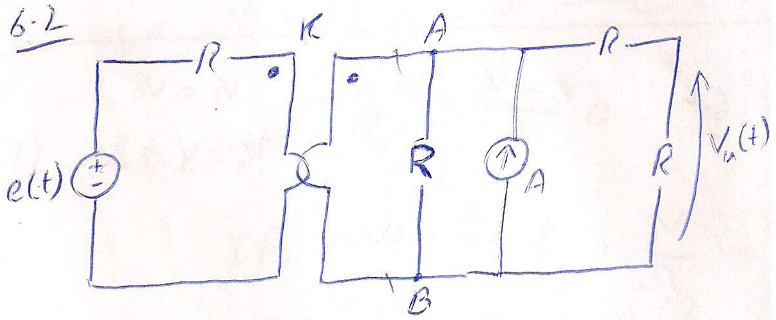
$$V_1 = KV$$
$$i_1 = -\frac{1}{K} i$$

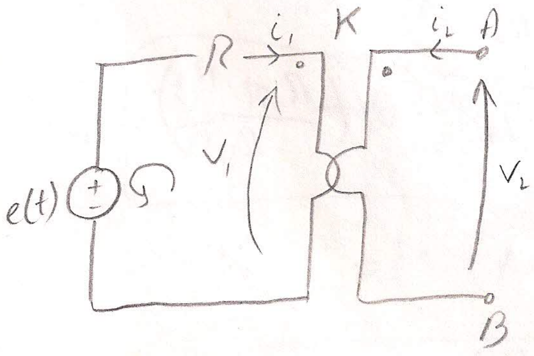
By KVL

$$V_1 + i_1 R = E$$
$$KV - \frac{Ri}{K} = E$$
$$V = \frac{E}{K} + \frac{R}{K^2} i$$

Hence
$$\left.\begin{aligned} R_{eq} &= \frac{R}{K^2}\ \Omega \\ V_{eq} &= \frac{E}{K}\ V \end{aligned}\right\}$$

6.2



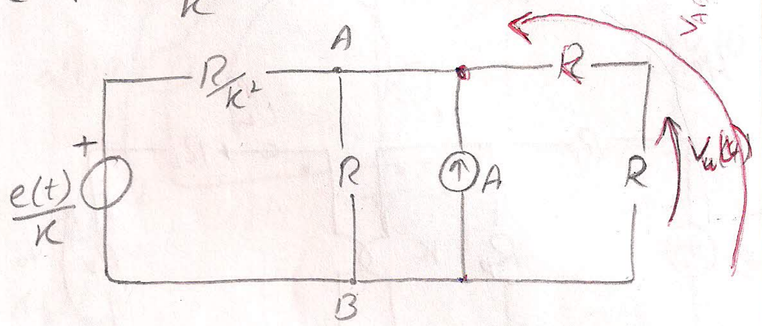


$$\begin{cases} V_1 = KV_2 \\ i_1 = -\frac{1}{K} i_2 \end{cases}$$

By KVL

$$V_1 + i_1 R = e(t)$$
$$KV_2 - \frac{R}{K} i_2 = e(t)$$
$$V_2 = \frac{e(t)}{K} + \frac{R}{K^2} i_2$$

$$\begin{cases} R_{eq} = \frac{R}{K^2} \\ V_{eq} = \frac{e(t)}{K} \end{cases}$$



By Millmann

$$V_u(t) = \frac{V_{AB}}{2} = \frac{\frac{e(t)}{K}\cdot\frac{K^2}{R} + A}{\frac{1}{R/K^2} + \frac{1}{R} + \frac{1}{R+R}} \cdot \frac{1}{2}$$

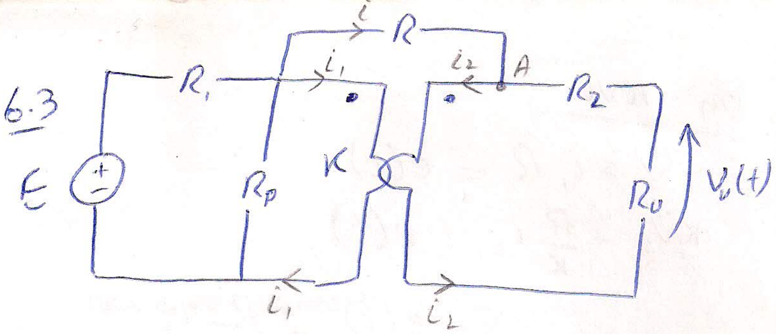
$$= \frac{K\cdot e(t) + RA}{\frac{K^2}{R} + \frac{1}{R} + \frac{1}{2R}} \cdot \frac{1}{2R}$$

$$= \frac{K\cdot e(t) + RA}{\frac{2K^2+2+1}{\cancel{2R}}} \cdot \frac{1}{\cancel{2R}}$$

$$\boxed{V_u(t) = \frac{Ke(t) + RA}{2K^2+3}\ V}$$

$$V_u(t) = \frac{R}{R+R} V_{AB} = \frac{V_{AB}}{2} = \underbrace{V_{AB}}_{\text{Millmann}} \cdot \frac{1}{2}$$

6.3

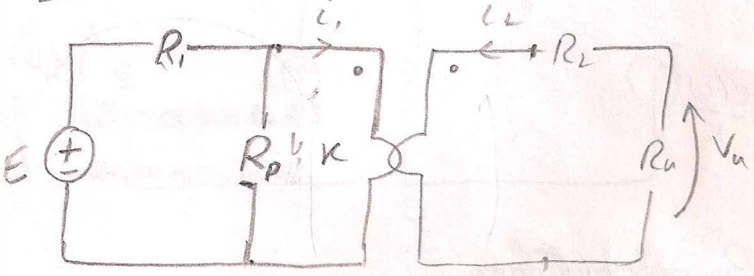


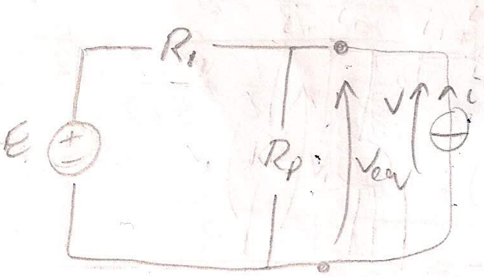
KCL @ A

$$-i + i_2 - i_2 = 0$$

$$i = 0$$

Hence





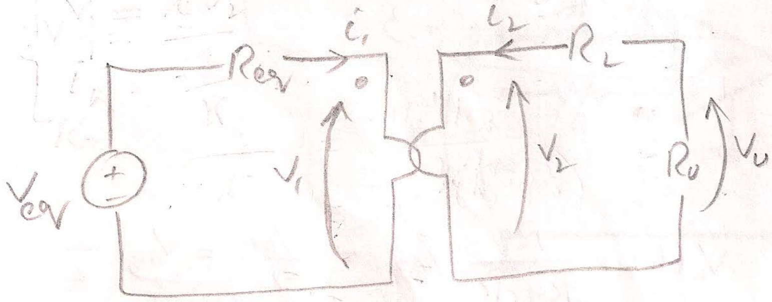
$$V = \frac{\frac{E}{R_1} + i_T}{\frac{1}{R_1} + \frac{1}{R_p}}$$

$$V = \frac{E + R_1 i_T}{\frac{R_1 + R_p}{R_p}}$$

$$V = \frac{R_p E}{R_1 + R_p} + \frac{R_1 R_p i_T}{R_1 + R_p}$$

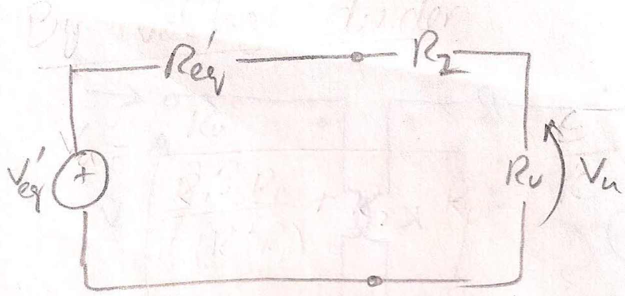
$$R_{eq} = \frac{R_1 R_p}{R_1 + R_p}$$

$$V_{eq} = \frac{R_p}{R_1 + R_p} E$$



$$V_{eq}' = \frac{V_{eq}}{K} = \frac{R_p E}{(R_1 + R_p) K}$$

$$R_{eq}' = \frac{R_{eq}}{K^2} = \frac{R_1 R_p}{K^2 (R_1 + R_p)}$$



$$V_u = \frac{R_u}{R_{eq}' + R_2 + R_u} \times V_{eq}'$$

$$= \frac{R_u}{\frac{R_1 R_p}{K^2 (R_1 + R_p)} + R_2 + R_u} \cdot \frac{R_p E}{(R_1 + R_p) K}$$

$$= \frac{K^2 \cancel{(R_1 + R_p)} R_u R_p E}{K \cancel{(R_1 + R_p)} \left[R_1 R_p + K^2 (R_1 + R_p)(R_2 + R_u) \right]}$$

$$\boxed{V_u = E \cdot \frac{R_u R_p K}{R_1 R_p + K^2 (R_p + R_1)(R_2 + R_u)}}$$

6.4

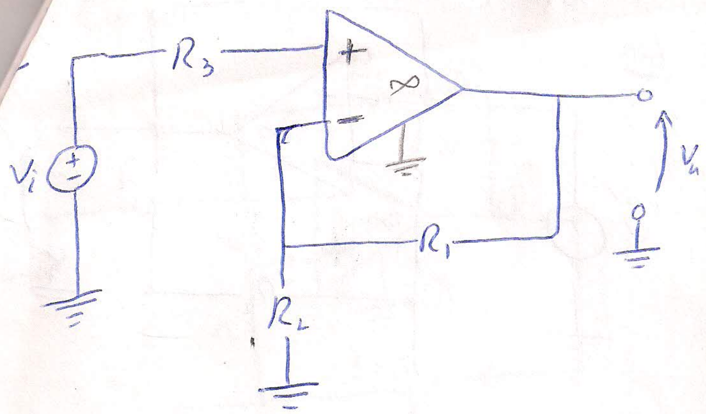
$$\frac{V_1 - V_i}{R_3} = 0 \Rightarrow V_1 = V_i$$

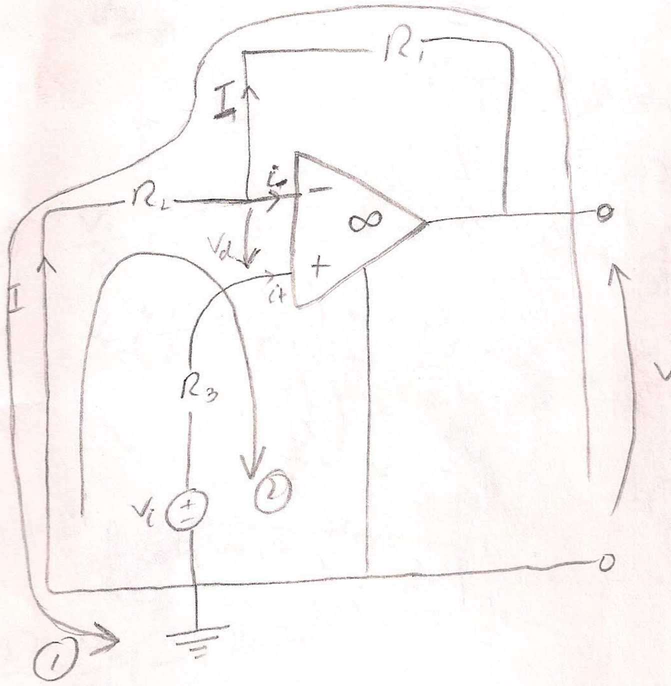
$$V_2 = V_1 = V_i$$

$$\frac{V_i}{R_2} + \frac{V_i - V_u}{R_1} = 0$$

$$V_i \left(\frac{R_1 + R_2}{R_1 R_2} \right) = \frac{V_u}{R_1}$$

$$\boxed{V_u = V_i \left(\frac{R_1 + R_2}{R_2} \right)}$$





KVL

1) $V_u + IR_1 + IR_2 = 0$ — (1)

2) $-IR_2 + \cancel{V_d}^{0} + \cancel{R_3} i_-^{0} - V_i = 0$

$IR_2 = -V_i$

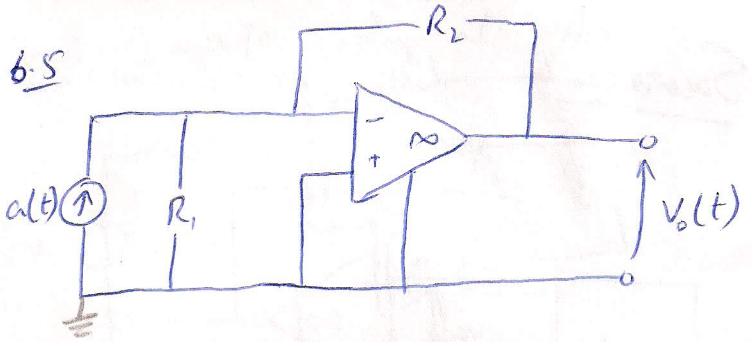
$I = \frac{-V_i}{R_2}$ put in (1)

$V_u - \frac{V_i}{R_2}R_1 - V_i\frac{R_2}{R_2} = 0$

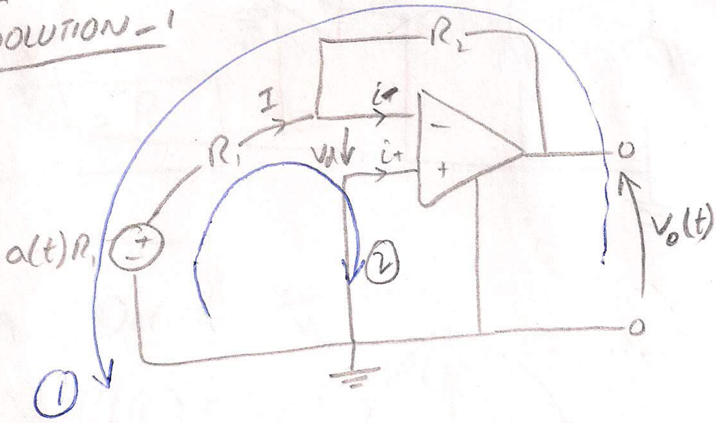
$V_u = V_i\left(1 + \frac{R_1}{R_2}\right)$

$\boxed{V_u = V_i\left(\frac{R_1+R_2}{R_2}\right)}$

6.5



Solution-1



KVL

1) $V_o + I(R_1+R_2) - aR_1 = 0$

2) $aR_1 - IR_1 + \cancel{V_d}^{0} = 0$

$aR_1 = IR_1$

$a = I$ put in (1)

$V_o + a(R_1+R_2-R_1) = 0$

$V_o(t) = -a(t)R_2$

$V_o(t) = -0.5\sin(500t)\cdot 10\cdot 10^{3}\cdot 10$

$\boxed{V_o(t) = -5\sin(500t)\,V}$

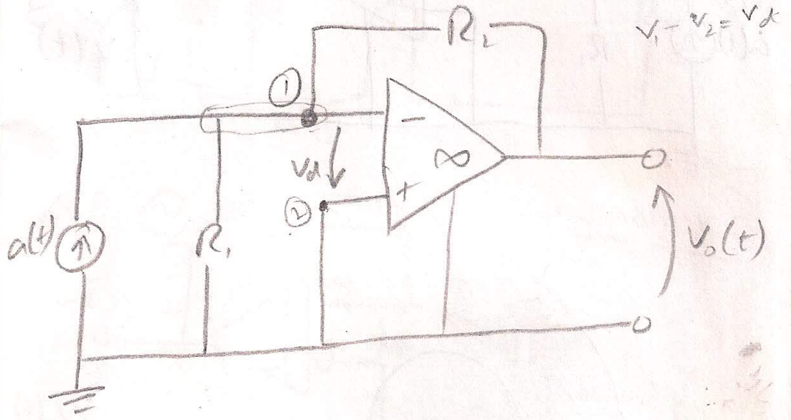
$V_1 = V_2 = 0$

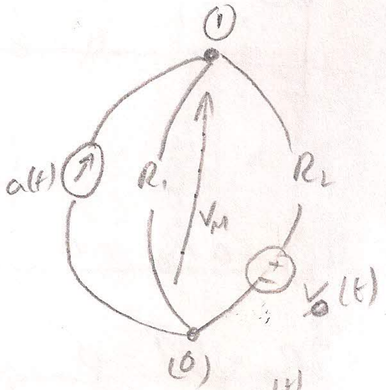
$-a(t) + \frac{V_1}{R_1} + \frac{V_1 - V_o(t)}{R_2} = 0$

$-\frac{V_o(t)}{R_2} = a(t)$

$\boxed{V_o(t) = -a(t)R_2}$

Solution_2 By Millmann





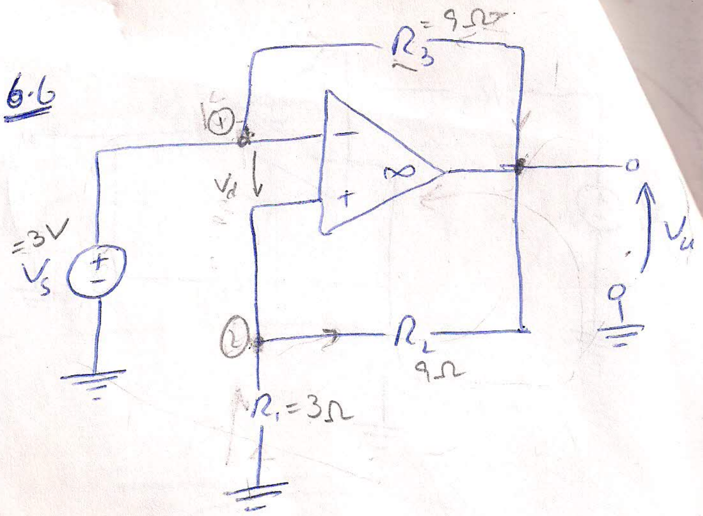
$$V_M = \frac{a(t) + \frac{V_o(t)}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$V_M = -V_d = 0$$

$$0 = a(t) + \frac{V_o(t)}{R_2}$$

$$\boxed{V_o(t) = -a(t)\, R_2}$$

6.6



$$V_1 = V_2 = V_s = 3V$$

2 KCL

$$\frac{V_2}{R_1} + \frac{V_2 - V_u}{R_2} = 0$$

$$\frac{3}{3} + \frac{3}{\cancel{9}\,3} - \frac{V_u}{9} = 0$$

$$\frac{V_u}{\cancel{9}\,3} = \frac{4}{3}$$

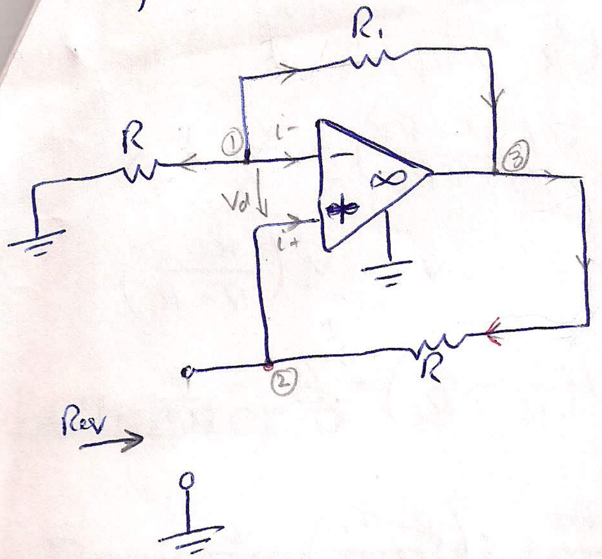
$$\boxed{V_u = 12V \quad Ans}$$

$$\frac{V_u}{R_2} = \frac{V_2}{R_1} + \frac{V_2}{R_2}$$

$$\frac{V_u}{R_2} = \frac{R_2V_2 + R_1V_2}{R_1R_2}$$

$$\boxed{V_u = V_s\left(\frac{R_1 + R_2}{R_1}\right) V} \quad Ans$$

Compute equivalent resistance



① $\frac{V_1}{R} + \frac{V_1 - V_3}{R_1} = 0$

$V_1\left(\frac{1}{R}+\frac{1}{R_1}\right) = \frac{V_3}{R_1}$

$V_1\left(\frac{R+R_1}{RR_1}\right) = \frac{V_3}{R_1}$

$V_3 = V_1\left(\frac{R+R_1}{R}\right)$ — ①

entering current

② $\frac{V_2}{R_{eq}} - \frac{V_2 - V_3}{R} = 0$

$V_1 = V_2$

$V_1\left(\frac{1}{R_{eq}} - \frac{1}{R}\right) = -\frac{V_3}{R}$

$V_1\left(\frac{R-R_{eq}}{R_{eq}R}\right) = -\frac{V_3}{R}$

$-V_1\left(\frac{R-R_{eq}}{R_{eq}}\right) = V_3$ — ②

comp ① & ②

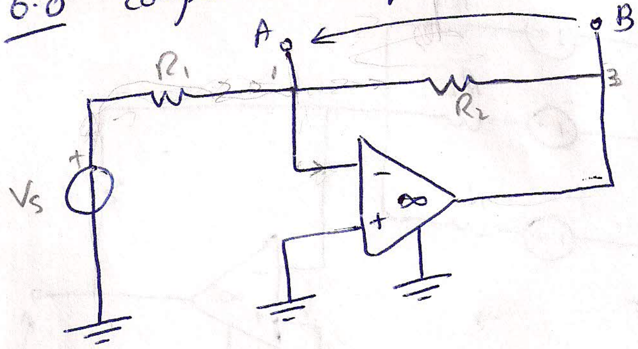
$\frac{R+R_1}{R} = -\left(\frac{R-R_{eq}}{R_{eq}}\right)$

$\frac{R+R_1}{R} = -\frac{R}{R_{eq}} + 1$

$\frac{R}{R_{eq}} = -\frac{R+R_1}{R} + 1 \Rightarrow \frac{R}{R_{eq}} = \frac{-\cancel{R}-R_1+\cancel{R}}{R}$

$\frac{R}{R_{eq}} = \frac{-R_1}{R} \Rightarrow \boxed{R_{eq} = -\frac{R^2}{R_1}\,\Omega}$ Ans

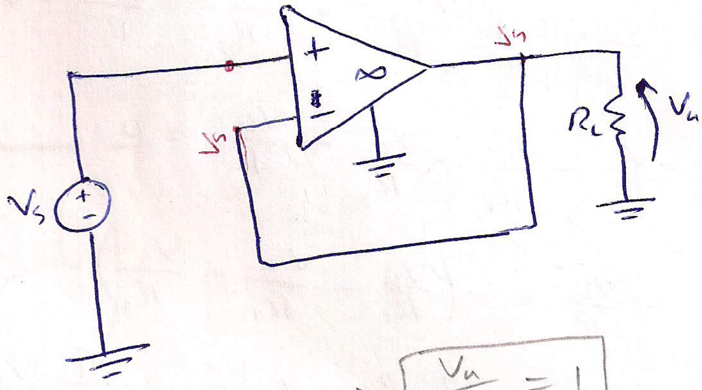
6.8 compute R_{eq} at AB



$\boxed{R_{eq} = R_2}$ Ans

6.9 Derive $\frac{V_u}{V_s} = ?$

$P(R_L) = ?$, $P(V_s) = ?$



a) $V_u = V_s \Rightarrow \boxed{\frac{V_u}{V_s} = 1}$

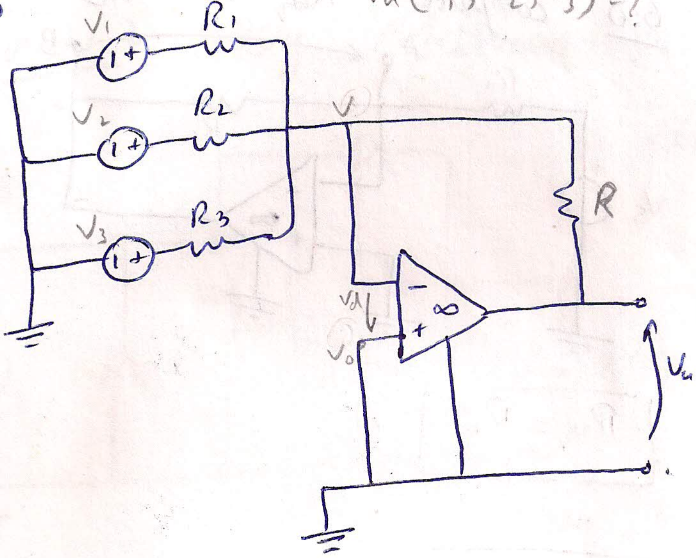
b) Power generated by the source will be the multiplication of source voltage and the current through it.

c) $P(R_L) = \frac{V_u^2}{R_L} = \frac{V_s^2}{R_L} = \frac{(3V)^2}{3\times10^3\Omega}$

$= \frac{9}{3\times10^3}$

$\boxed{P(R_L) = 3\,mW}$ Ans

6.10 $V_u(V_1, V_2, V_3) = ?$



$V_1 = V_0 = 0 \quad (V_0 = 0)$

$V = 0$

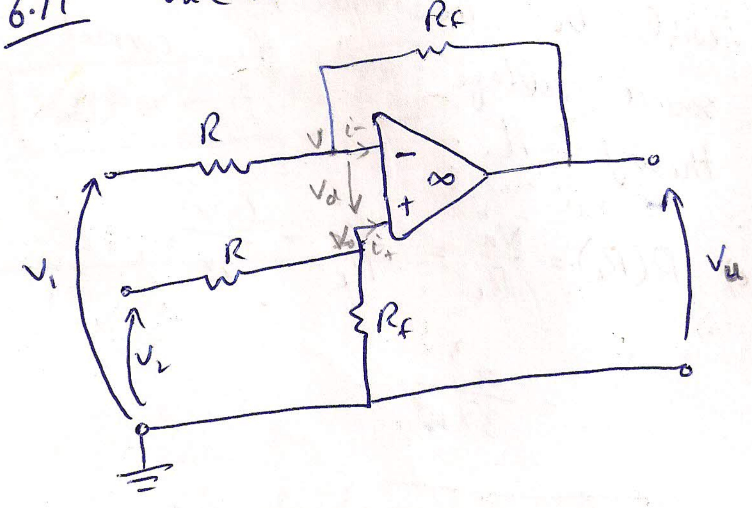
$$① \quad \frac{V - V_1}{R_1} + \frac{V - V_2}{R_2} + \frac{V - V_3}{R_3} + \frac{V - V_u}{R} = 0$$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} - \frac{V_3}{R_3} - \frac{V_u}{R} = 0$$

$$\frac{V_u}{R} = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

$$V_u = -\left(V_1\frac{R}{R_1} + V_2\frac{R}{R_2} + V_3\frac{R}{R_3}\right) V$$

6.11 $V_u(V_1, V_2) = ?$



$V = V_0$

$$① \quad \frac{V - V_1}{R} + \frac{V - V_u}{R_f} = 0$$

$$② \quad \frac{V_0 - V_2}{R} + \frac{V_0}{R_f} = 0$$

$V = V_0$

$$V\left(\frac{1}{R} + \frac{1}{R_f}\right) = \frac{V_2}{R}$$

$$V\left(\frac{R + R_f}{R R_f}\right) = \frac{V_2}{R}$$

$$V = V_2\left(\frac{R_f}{R + R_f}\right) \text{ put in ①}$$

$$① \quad V\left(\frac{1}{R} + \frac{1}{R_f}\right) - \frac{V_1}{R} = \frac{V_u}{R_f}$$

$$V_2\left(\frac{\cancel{R_f}}{\cancel{R + R_f}}\right)\left(\frac{\cancel{R + R_f}}{R \cancel{R_f}}\right) - \frac{V_1}{R}$$

$$\frac{V_2}{R} - \frac{V_1}{R} = \frac{V_u}{R_f}$$

$$\boxed{V_u = \frac{R_f}{R}(V_2 - V_1) \; V}$$

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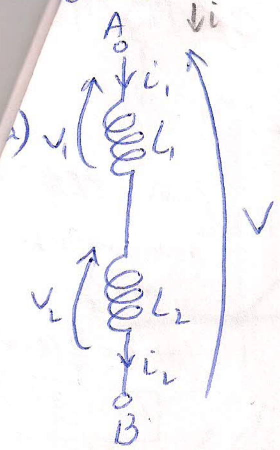
etermine characteristic equations

a)



As $V_1 = L_1 \frac{di_1}{dt}$

$V_2 = L_2 \frac{di_2}{dt}$

$i = i_1 = i_2$

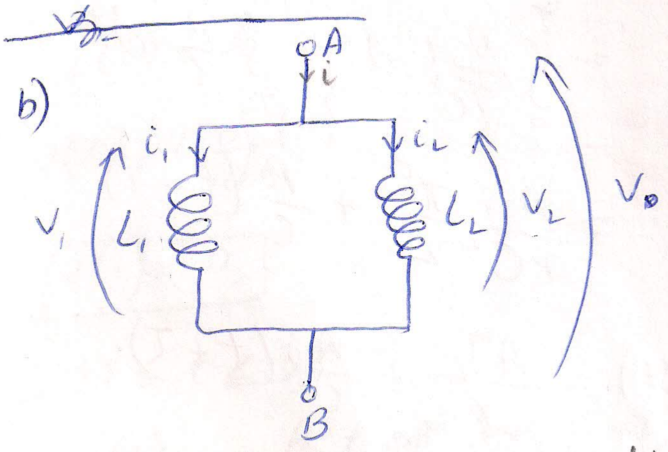
$V = V_1 + V_2$

$V = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

$\boxed{V = \frac{di}{dt}(L_1 + L_2)}$ $(i_1 = i_2 = i)$

---

b)



$i = i_1 + i_2$

$V = V_1 = V_2$

$V_1 = L_1 \frac{di_1}{dt}$

$V_2 = L_2 \frac{di_2}{dt}$

$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

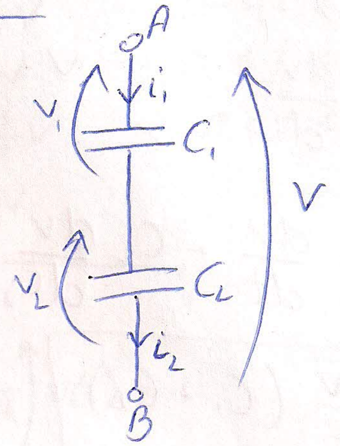
$\frac{di}{dt} = \frac{V}{L_1} + \frac{V}{L_2}$

$\frac{di}{dt} = V\left(\frac{L_1 + L_2}{L_1 L_2}\right)$

$\boxed{V = \left(\frac{L_1 L_2}{L_1 + L_2}\right)\frac{di}{dt}}$

8-2

a)



$i_1 = C_1 \frac{dv_1}{dt}$

$i_2 = C_2 \frac{dv_2}{dt}$

$i = i_1 = i_2$

$V = V_1 + V_2$

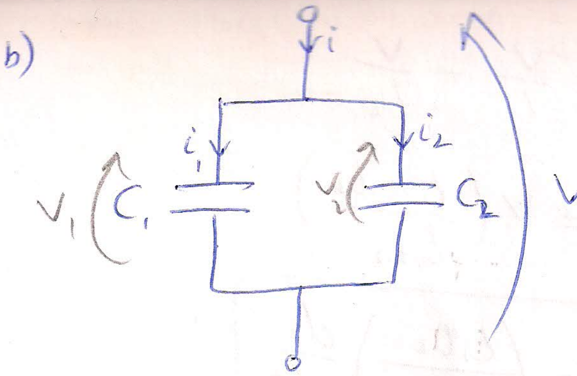
$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$

$\frac{dv}{dt} = \frac{i}{C_1} + \frac{i}{C_2}$

$\frac{dv}{dt} = i\left(\frac{C_1 + C_2}{C_1 C_2}\right)$

$\boxed{i = \frac{C_1 C_2}{C_1 + C_2}\frac{dv}{dt}}$

b)



$i_1 = C_1 \frac{dv_1}{dt}$ $\quad$ $i = i_1 + i_2$

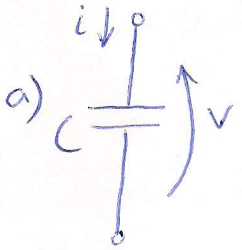
$i_2 = C_2 \frac{dv_2}{dt}$ $\quad$ $V = V_1 = V_2$

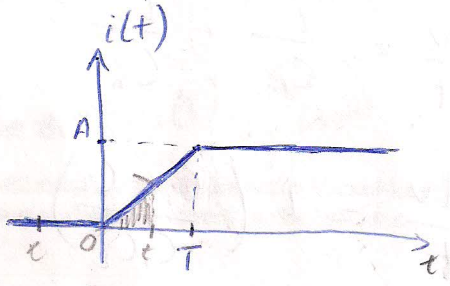
$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$

$\boxed{i = \frac{dv}{dt}(C_1 + C_2)}$ $\quad$ A_2

8.3 Determine the voltage

a)





$i(t) = C \frac{dv(t)}{dt}$

$v(t) = \int \frac{i(t)}{C} dt$

$-v(t) = 0 \quad t<0$

$-v(t) = \int_0^t \frac{i(t)}{C} dt \quad 0 \le t < T$

$= \frac{1}{C} \int_0^t i(t) dt$

$(0,0)$, (T,A)

$(0-A) = m(0-T)$

$m = \frac{A}{T}$

$i(t) - 0 = \frac{A}{T}(t-0)$

$\boxed{i(t) = \frac{A}{T}t}$

$i(t) = \frac{A}{T}t$

$= \frac{A}{TC} \int_0^t t.dt$

$= \frac{A}{TC} \cdot \frac{t^2}{2}$

$= \frac{A}{CT} \cdot \frac{t^2}{2}$

$-v(t) = \int_T^{+\infty} \frac{i(t)}{C} dt \quad t \ge T$

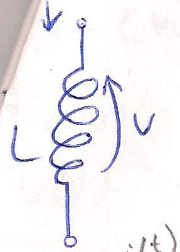
$= \int_0^T \frac{i(t)}{C} dt + \int_T^t \frac{i(t)}{C} dt$

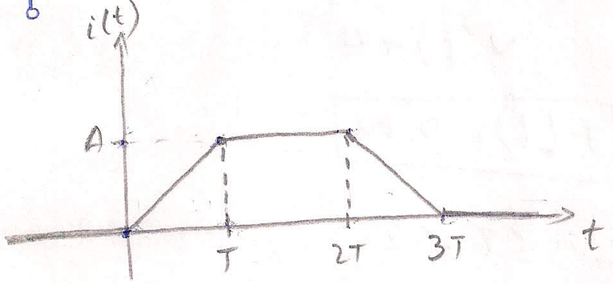
$= \int_0^T \frac{A}{TC} t \, dt + \int_T^t \frac{A}{C} dt$

$= \frac{A}{TC} \cdot \frac{T^2}{2} + \frac{A}{C}(t-T)$

$v(t) = \frac{AT}{2C} + \frac{A}{C}(t-T)$

$$v(t) = \begin{cases} 0 & t<0 \\ \frac{A}{CT}\cdot\frac{t^2}{2} & 0 \le t < T \\ \frac{AT}{2C} + \frac{A}{C}(t-T) & t \ge T \end{cases}$$





$v(t) = L\dfrac{di(t)}{dt}$

- $t<0$

$\boxed{v(t) = 0\ V}$

- $0 \le t < T$

$i(t) = \dfrac{A}{T} t$

$v(t) = \dfrac{A}{T} L \dfrac{d}{dt}(t)$

$\boxed{v(t) = L\dfrac{A}{T}\ V}$

$(0,0) \qquad (T, A)$

$(0-A) = m(0-T)$

$m = \dfrac{A}{T}$

$i(t) - 0 = \dfrac{A}{T}(t-0)$

$\boxed{i(t) = \dfrac{A}{T} t}$

$m(x_1 - x_2) = (y_1 - y_2)$

$m(2T-3T) = A-0$

$m = -\dfrac{A}{T}$

- $T \le t < 2T$

$i(t) = A$

$v(t) = L\dfrac{d}{dt}(A)$

$\boxed{v(t) = 0}$

- $2T \le t \le 3T$

$i(t) = -\dfrac{A}{T} t \cancel{+3A}$ (-ve slope)

$v(t) = L\dfrac{d}{dt}\left(-\dfrac{A}{T} t\right)$

$\boxed{v(t) = -\dfrac{A}{T} L\ \ V}$

$(2T, A) \qquad (3T, 0)$

$A = m(-T)$

$m = -\dfrac{A}{T}$

$\cancel{i(t) = 2T - \frac{A}{T}(t-A)}$

$i(t) - A = -\dfrac{A}{T}(t-2T)$

$i(t) = -\dfrac{A}{T}t + 2T\dfrac{A}{T} + A$

$\boxed{i(t) = -\dfrac{A}{T}t + 3A}$

- $3T \le t$

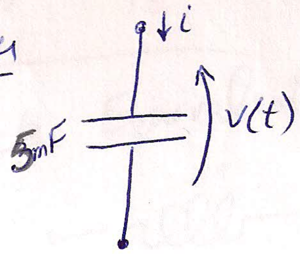
$i(t) = 0$

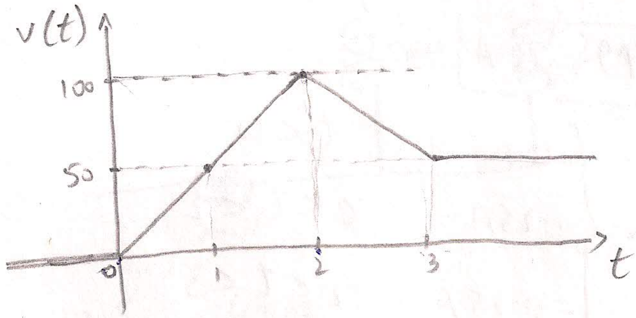
$\boxed{v(t) = 0\ V}$

Hence

$$v(t) = \begin{cases} 0\ V & t < 0 \\ L\dfrac{A}{T}\ V & 0 \le t < T \\ 0\ V & T \le t < 2T \\ -L\dfrac{A}{T}\ V & 2T \le t < 3T \\ 0\ V & t \ge 3T \end{cases}$$

8.4





$i(t) = C\dfrac{d}{dt} v(t)$

- $t<0$, $v(t) = 0$

$\boxed{i(t) = 0\ A}$

- $0 \le t < 2$

$v(t) = 50t$

$i(t) = 5 \times 10^{-3} \times 50 \times \underbrace{\dfrac{d}{dt}(t)}_{=1}$

$\boxed{i(t) = 0.25\ A}$

– $2 \le t < 3$

$(100, 2), (50, 3)$

$(100-50) = m(2-3)$

$m = -50$

$\boxed{(v-100) = -50(t-2)}$ —— ?

$v(t) = -50t + 100 + 100$

$v(t) = -50t + 200$

$i(t) = 5\times10^{-3} \dfrac{d}{dt}(-50t + 200)$

$= -5 . 50 \times 10^{-3}$ A

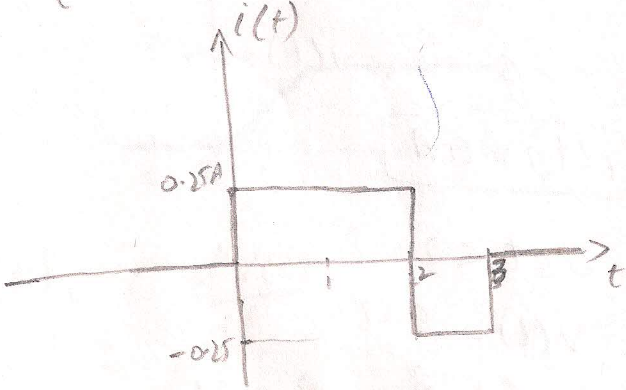
$\boxed{i(t) = -0.25 \text{ A}}$

– $t \ge 3$

$v(t) = 50$ V

$\boxed{i(t) = 0 \text{ A}}$

$$i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 0.25\text{A} & 0 \le t < 2 \\ -0.25 \text{A} & 2 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$



$P(t) = v(t) i(t)$

$P(t) = v(t)\, C \dfrac{dv(t)}{dt}$

– $t < 0$

$v(t) = 0$

$\boxed{P(t) = 0 \text{ W}}$

– $0 \le t < 2$

$v(t) = 50t$

$P(t) = 50t \times 5m \times 50$

$\boxed{P(t) = 12.5t \text{ W}}$

– $2 \le t < 3$

$(100, 2), (50, 3)$

$(100-50) = m(2-3) \Rightarrow m = -50$

$v(t) - 100 = -50(t-2)$

$v(t) = -50t + 200$

$P(t) = 5m(-50t + 200) \dfrac{d}{dt}(-50t + 200)$

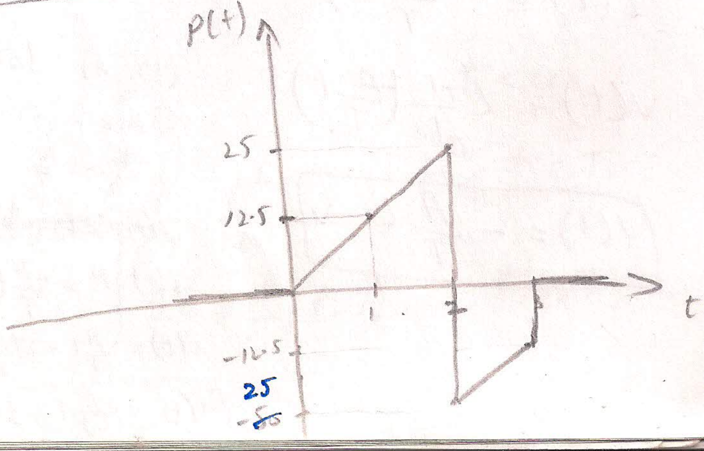
$= 250\times10^{-3}(t-4) \times -50$

$\boxed{P(t) = 12.5(t-4) \text{ W}}$

– $t \ge 3$

$v(t) = 50$ V

$\boxed{P(t) = 0 \text{ W}}$



$\frac{1}{2} C V^2(t)$

$t < 0$

$v(t) = 0$

$\boxed{U = 0\,J}$

$- \; 0 \leq t < 2$

$v(t) = 50t$

$U = \frac{1}{2} 5m \; 2500 t^2$

$\boxed{U = 6.25 t^2 \, J}$

$- \; 2 \leq t < 3$

$(100, 2) \quad (50, 3)$

$(100 - 5) = m(2 - 3) \Rightarrow m = -50$

$V(t) - 100 = -50(t - 2)$

$V(t) = -50t + 200 \, V$

$U(t) = \frac{1}{2} 5m . 2500 (t - 4)^2$

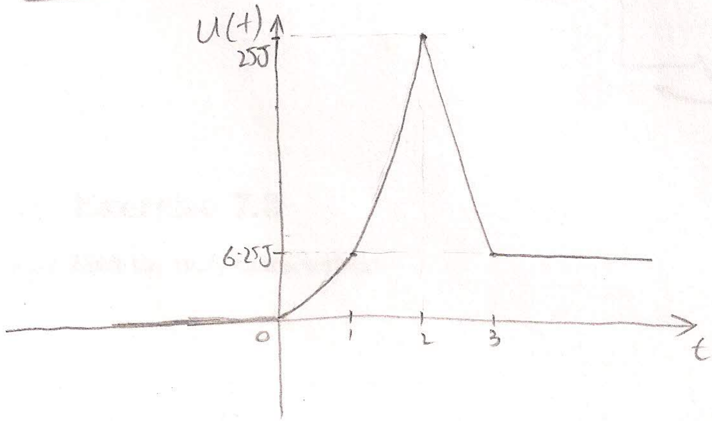
$\boxed{U(t) = 6.25 (t - 4)^2 \, J}$

$- \; t \geq 3$

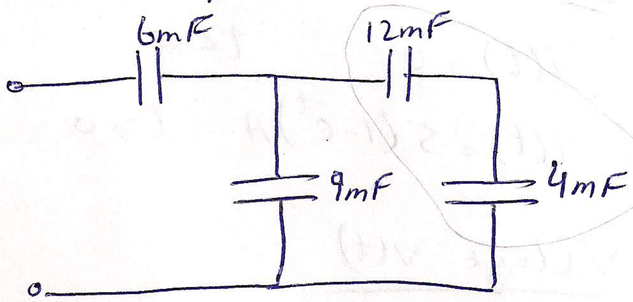
$v(t) = 50\,V$

$U(t) = \frac{1}{2} 5m \; 2500$

$\boxed{U(t) = 6.25\,J}$



8.5 Derive equivalent capacitance.

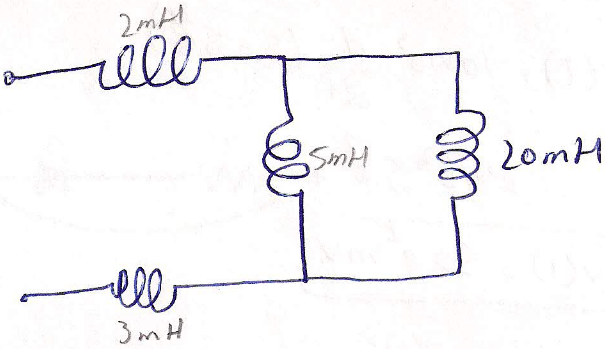


$\frac{12 . 4}{16} = 3\,mF$

$3 + 9 = 12\,mF$

$C_{eq} = \frac{12 . 6}{18} = 4\,mF$ Ans

8.6 Equivalent inductance



$\frac{20 . 5}{25} = 4\,mH$

$L_{eq} = 4 + 2 + 3 = 9\,mH$ Ans

8.7 $L = 10mH$

$i(t) = 0 \qquad t<0$

$i(t) = 5(1-e^{-t})A \qquad t \geq 0$

Voltage $v(t)$

- $t<0$

$i(t) = 0$

$v(t) = L\dfrac{di(t)}{dt}$

$\boxed{v(t) = 0}$

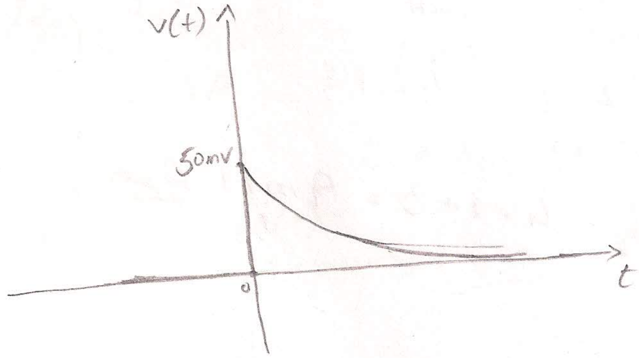
- $t \geq 0$

$i(t) = 5(1-e^{-t})A$

$v(t) = 10\times10^{-3}\dfrac{d}{dt}(5-5e^{-t})$

$= 10\times10^{-3}\, 5e^{-t}$

$\boxed{v(t) = 50e^{-t}\, mV}$



Power $P(t) = v(t)\, i(t)$

- $t<0$

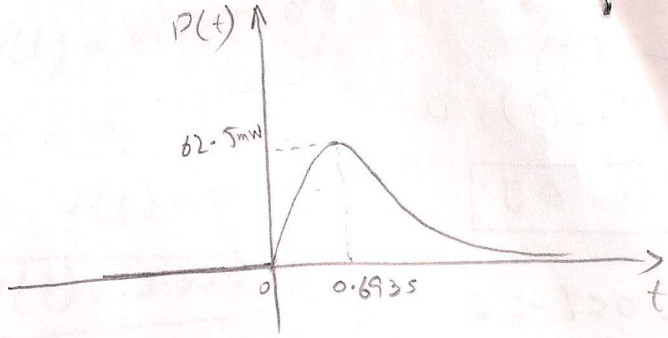
$i(t) = 0 \quad , \quad v(t) = 0$

$P(t) = 0$

- $t \geq 0$

$i(t) = 5(1-e^{-t})A \qquad v(t) = 50e^{-t}\, mV$

$P(t) = 250(1-e^{-t})e^{-t}\, mW$



Energy $(U = \frac{1}{2}L i^2(t))$

- $t<0$

$i(t) = 0$

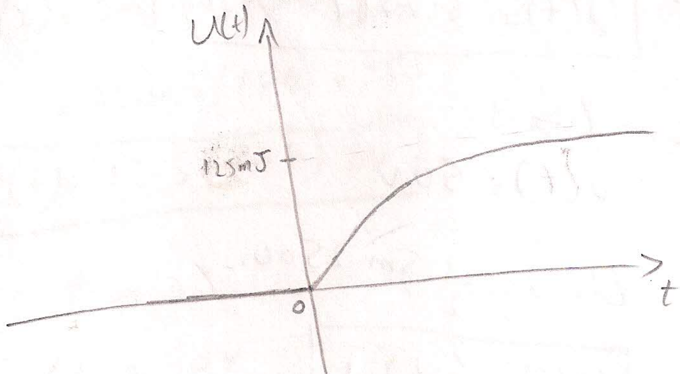
$\boxed{U(t) = 0\ J}$

- $t \geq 0$

$i(t) = 5(1-e^{-t})A$

$U = \frac{1}{2}\, 10\times10^{-3}\, 25(1-e^{-t})^2\, J$

$U = 125(1-e^{-t})^2\, mJ$



$\tau_c = R_{eq} C$

$\tau_L = \frac{L}{R_{eq}}$

$v_c(t) = [v(0^-) - v_{c\infty}] e^{-\frac{t-t_0}{\tau}} + v_{c\infty} \quad (t \geq 0)$

$i_L(t) = [i_L(0^-) - i_{L\infty}] e^{-\frac{t-t_0}{\tau}} + i_{L\infty} \quad (t \geq 0)$

$(v_0 - v) e^{-\frac{(t-t_0)}{\tau}} + v$

Chapter 9

First order circuits

Exercise 9.1

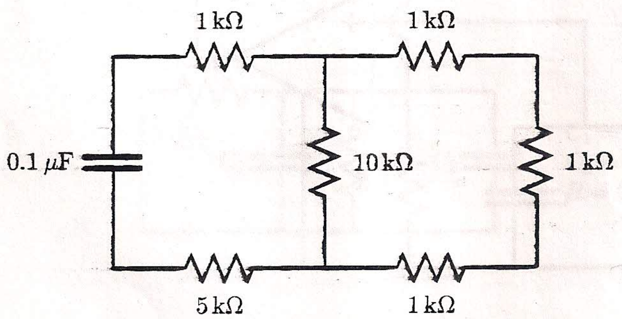
Find the waveform $v(t)$ that satisfies the following differential equation:

$$\frac{dv(t)}{dt} + 10\,v(t) = 0$$

with $v(0) = 5\,\text{V}$.

Exercise 9.2

Find the time constant of the circuit.



Exercise 9.3

Find the time constant τ of the circuit.

Find the waveform $v(t)$ that satisfies the following differential equation.

$$\frac{dv(t)}{dt} + 10\,v(t) = 0$$

As $\quad \frac{dx}{dt} + \frac{dx}{\tau} = \frac{g_{eq}}{\tau}$

$$\frac{dv(t)}{dt} = -10\,v(t)$$

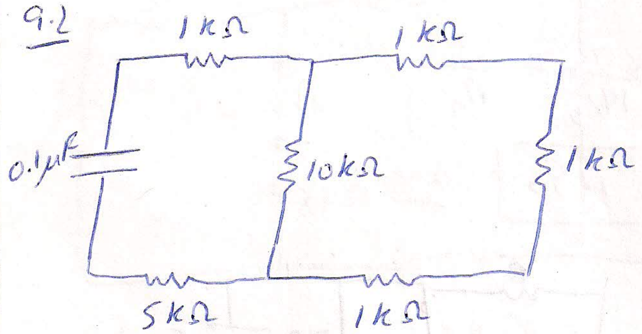
$$\int_0^t \frac{dv(t)}{v(t)} = \int_0^t -10\,dt$$

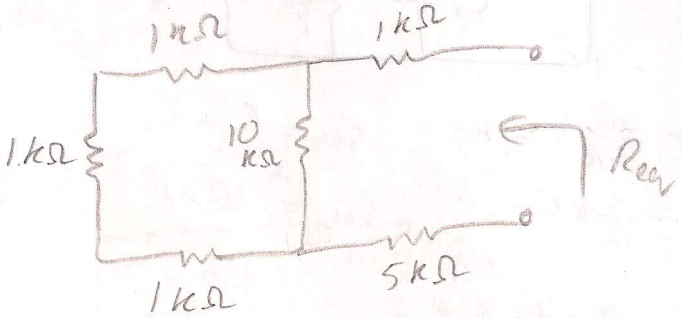
$$\ln v(t)\Big|_0^t = -10t$$

$$\ln \frac{v(t)}{v(0)} = -10t \Rightarrow \frac{v(t)}{v(0)} = e^{-10t}$$

$$\boxed{v(t) = 5e^{-10t}}$$

9.2





$$R_{eq} = 3 \,\|\, 10 + 6$$

$$= \frac{30}{13} + 6$$

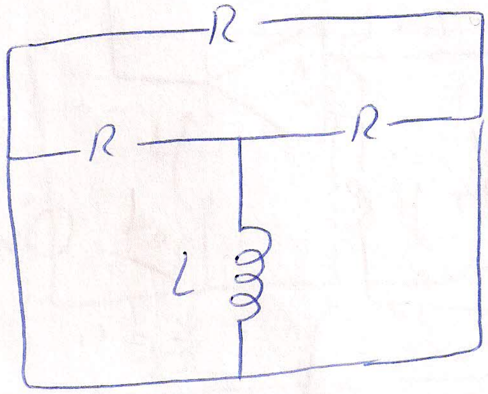
$$R_{eq} = 8.30\,k\Omega$$

$$\tau = R_{eq}C$$

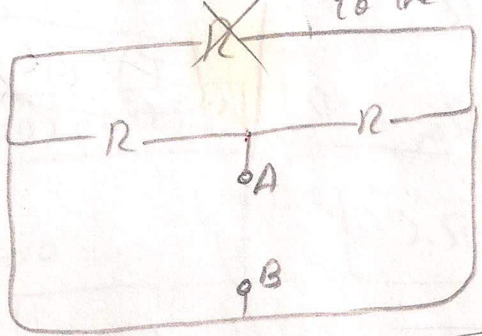
$$= 8.30 \times 0.1 \times 10^{-6} \times 10^{3}$$

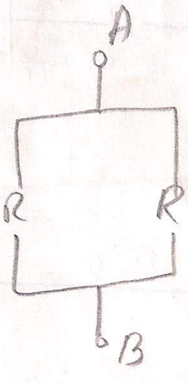
$$\boxed{\tau = 0.83\,ms}$$

9.3 $\quad \tau = ?$ $\qquad R = 1\Omega$, $L = \frac{1}{2}H$

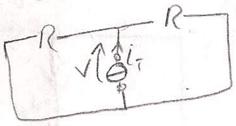


because it connects to the same terminal B.





OR



$$V = \frac{i_T}{\frac{2}{R}}$$

$$V = \frac{1}{2} i_T$$

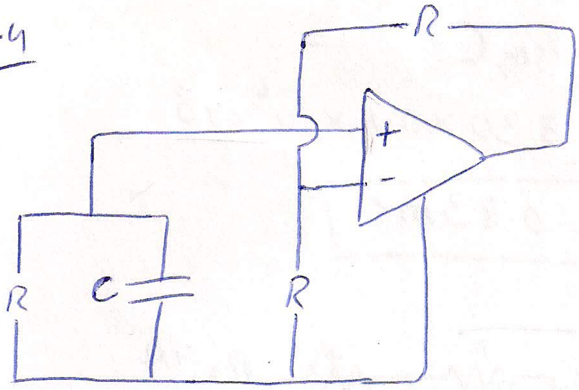
$$R_{eq} = \frac{1}{2}$$

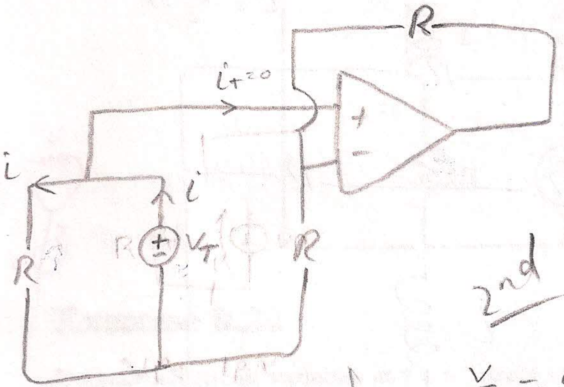
$$R_{eq} = \frac{1}{1+1} = \frac{1}{2}$$

$$\tau = \frac{L}{R_{eq}}$$

$$\boxed{\tau = 1\,s}$$

9.4





$R_{eq} = \frac{V_T}{i}$

$\frac{V_T}{i} = R$

so, $R_{eq} = R$

$$\boxed{\tau = R.C}$$

2nd

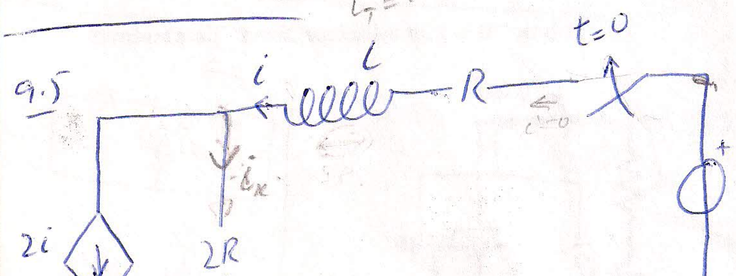
$\frac{V}{R} = i_T$

$V = Ri_T + 0$

$R_{eq} = R$

$$\boxed{\tau = R.C}$$

9.5

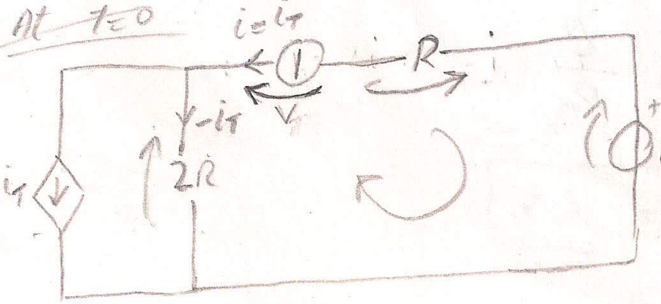


AT t=0

$i_x + 2i - i = 0$

$i_x = -i$

At t=0



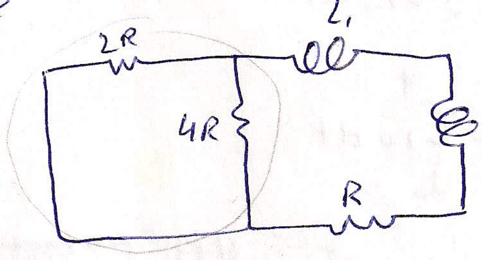
$i_T R - 2Ri_T - E_0 - V_T = 0$

$V_T = -Ri_T - E_0$

R_{eq} V_{eq}

$$\boxed{\tau = \frac{-L}{R}}$$

9.6 Find τ



$\frac{8R^2}{6R} = \frac{4}{3}R$

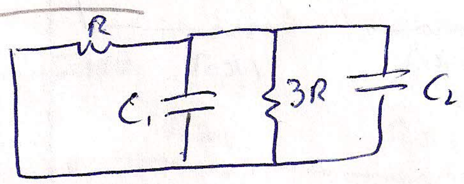
$R_{eq} = \frac{4}{3}R + R$ $\qquad L_{eq} = L_1 + L_2$

$R_{eq} = \frac{7}{3}R = \frac{14}{3}$ $\qquad L_{eq} = 7H$

$\tau = \frac{7}{14/3} = \frac{21}{14}$

$\boxed{\tau = 1.5s}$ Ans

9.7

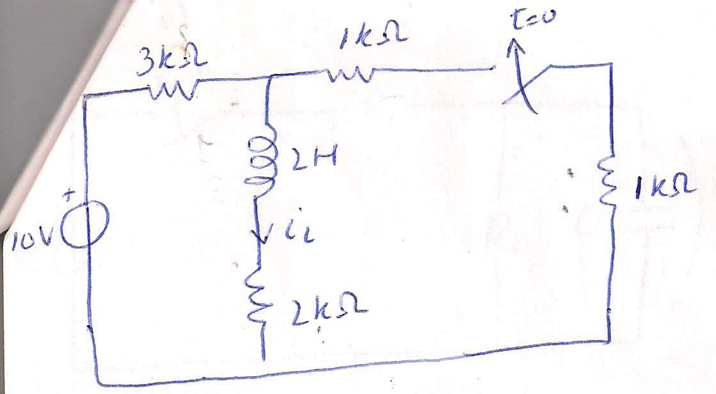


$R_{eq} = \frac{3R^2}{4R} = \frac{3}{4}R$ $\qquad C_{eq} = C_1 + C_2$

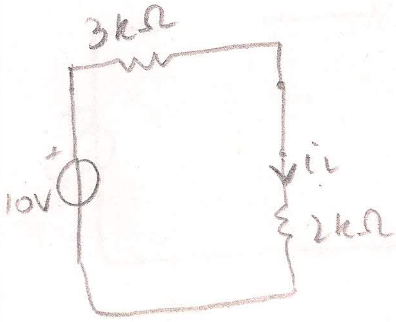
$R_{eq} = \frac{3}{2}k\Omega$ $\qquad C_{eq} = 10\mu F$

$\tau = \frac{3}{2} \times 10^5\ k.\mu s$

$\boxed{\tau = 15ms}$ Ans



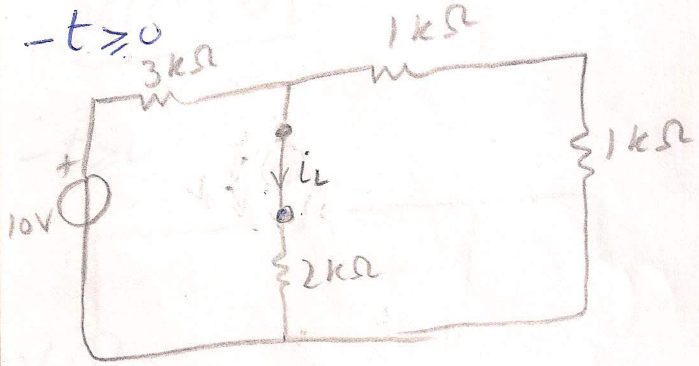
−t<0



$i_L(0) = \frac{10}{5\times10^3}$

$\boxed{i_L(0) = 2\,mA}$ ✓

−t≥0

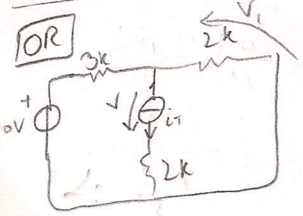


$R_{eq} = \frac{6}{5} + 2 = \frac{16}{5} = 3.2\,k\Omega$

$\tau = \frac{2}{3.2k}$

$\boxed{\tau = 0.625\,ms}$ Ans

OR



$V_1 = \frac{-\frac{10}{3k} + i_T}{\frac{1}{3k} + \frac{1}{2k}} = \frac{\frac{-10+3ki_T}{3k}}{\frac{5}{2.6k}}$

$V_1 = (-10 + 3ki_T)\frac{2}{5}$

$V_1 = -4 + \frac{6}{5}k\,i_T$

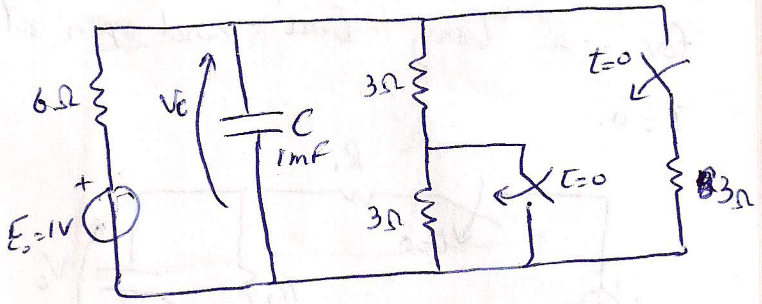
$V = -V_1 + 2k\,i_T$

$V = -4 + \frac{6}{5}k\,i_T + 2k\,i_T$

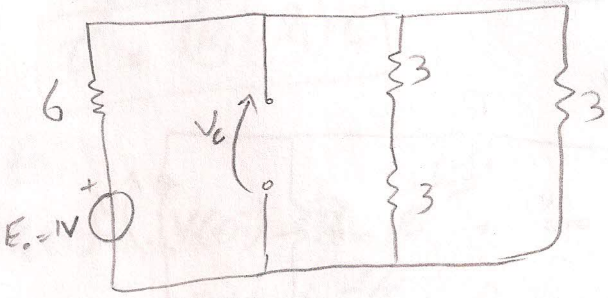
$V = -4 + \frac{16}{5}k\,i_T$

$\boxed{R_{eq} = 3.2k}$

9.9 $\tau = ?$, $V_c(0) = ?$



−t<0

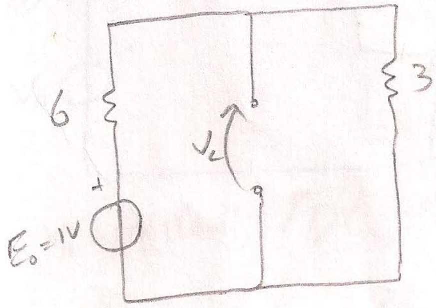


$V_c(0) = \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{6} + \frac{1}{3}} = \frac{1/6}{\frac{1+1+2}{6}}$

$= \frac{1}{4}$

$\boxed{V_c(0) = 0.25\,V}$

−t≥0



$R_{eq} = \frac{18}{9} = 2\,\Omega$

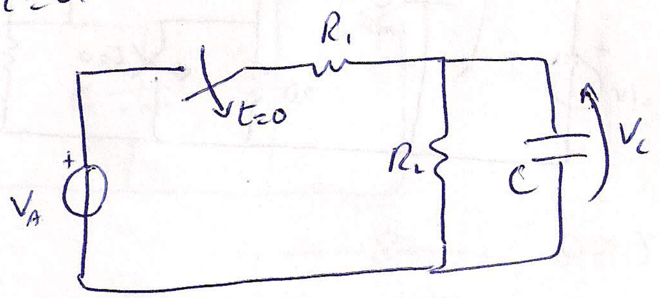
$\tau = R_{eq}C$

$\tau = 2 \cdot 1m$

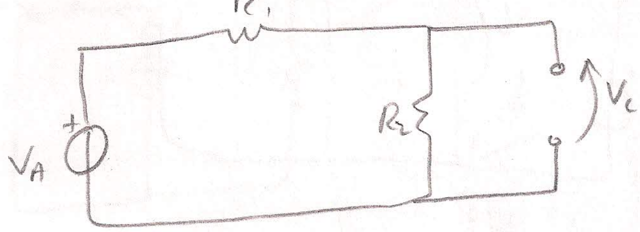
$\boxed{\tau = 2\,ms}$

9.10 9.10

ii) If switch remains closed for a "Long Time" and open at $T=0$.

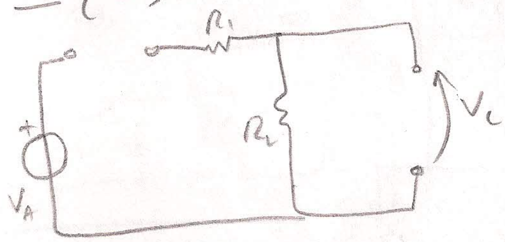


$- t<0$



a) $\boxed{V_c(0) = \left(\frac{R_2}{R_1+R_2}\right) V_A}$

$- t \to \infty$



b) $\boxed{V_c(\infty) = 0V}$

c) $\tau = R_{eq} C$

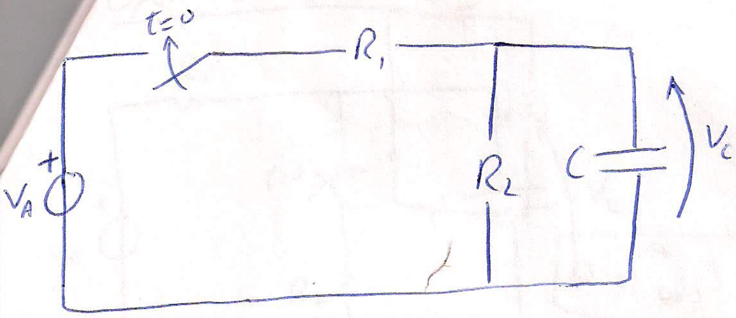
$R_{eq} = R_2$

$\boxed{\tau = R_2 . C}$

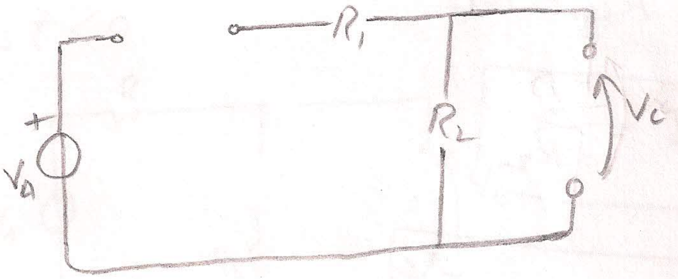
d) $V_c(t) = \left(V_c(0) - V_c(\infty)\right) e^{-\frac{t-t_0}{\tau}} + V_c(\infty)$

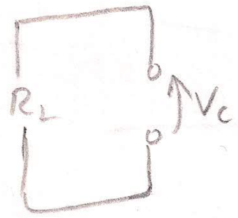
$\boxed{V_c(t) = \left(\frac{R_2 V_A}{R_1+R_2}\right) e^{-t/\tau}}$

i)



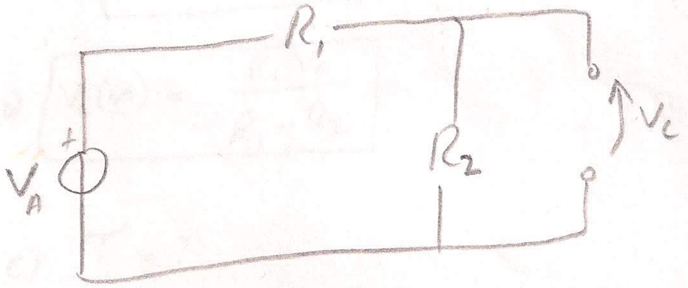
- $t < 0$



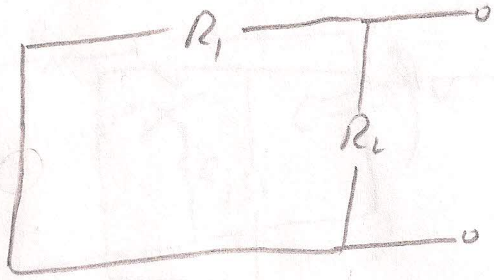


$$\boxed{V_c(0^-) = 0}$$

- $t \geq 0$



$$\boxed{V_c = \frac{R_2}{R_1+R_2} V_A}$$



$$R_{eq} = R_1 \parallel R_2$$

$$\boxed{\tau = (R_1 \parallel R_2) C}$$

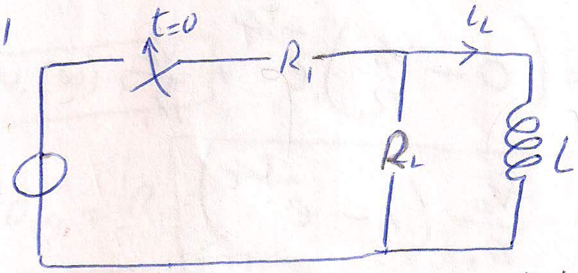
As

$$V_c(t) = (V(0^-) - V_{c\infty}) e^{-\frac{t-t_0}{\tau}} + V_{c\infty}$$

$$= -\left(\frac{R_2 V_A}{R_1+R_2}\right) e^{-t/\tau} + \frac{R_2 V_A}{R_1+R_2}$$

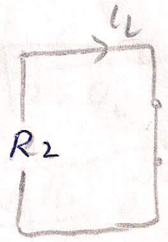
$$\boxed{V_c(t) = \frac{R_2 V_A}{R_1+R_2}\left(1 - e^{-t/\tau}\right)}$$

9.11
(i)



V_A = constant (DC steady state)

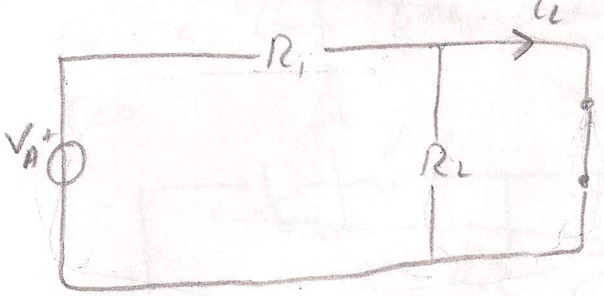
- $t < 0$



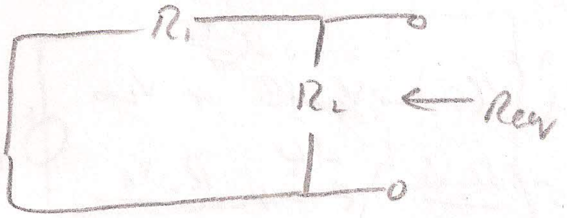
a)

$$\boxed{i_L(0) = 0\,A}$$

$-t \geq 0$



b) $\boxed{i_{L\infty} = \frac{V_A}{R_1}}$



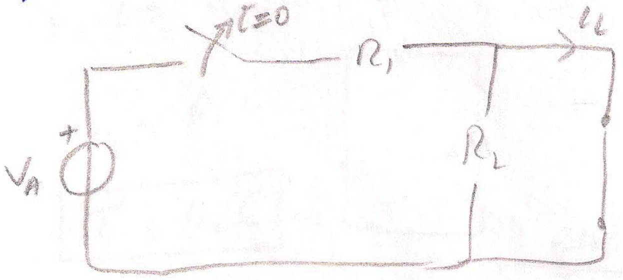
$R_{eq} = R_1 \parallel R_2$

$$\boxed{\tau = \frac{L}{R_1 \parallel R_2}}$$

$$i_L(t) = \left(0 - \frac{V_A}{R_1}\right) e^{-t/\tau} + \frac{V_A}{R_1}$$

$$\boxed{i_L(t) = \frac{V_A}{R_1}\left(1 - e^{-t/\tau}\right)}$$

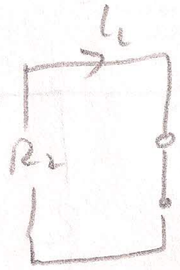
2) closed_to_open



$-t < 0$

$$\boxed{i_L(0^-) = \frac{V_A}{R_1}}$$

$-t \geq 0$



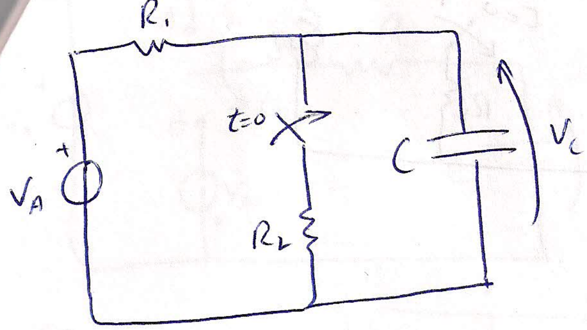
$$\boxed{i_{L\infty}^{+} = 0}$$

$R_{eq} = R_2$

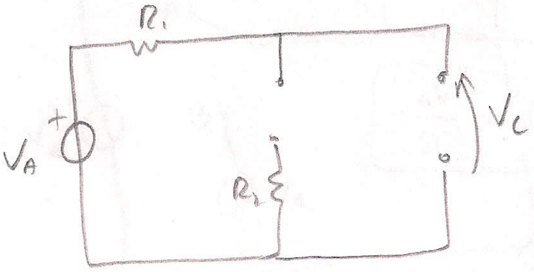
$$\boxed{\tau = \frac{L}{R_2}}$$

$$\boxed{i_L(t) = \frac{V_A}{R_1} e^{-t/\tau}}$$

Open-to-closed

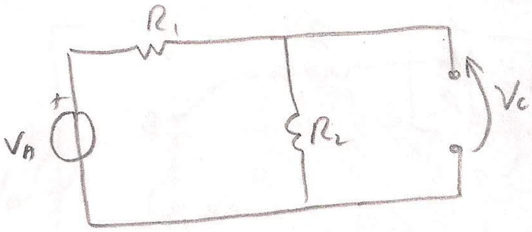


- $t<0$



a) $\boxed{V_c(0) = V_A}$

- $t \to \infty$



b) $\boxed{V_c(\infty) = \dfrac{R_2 V_A}{R_1+R_2}}$

c) $\tau = ?$

$R_{eq} = R_1 \parallel R_2$

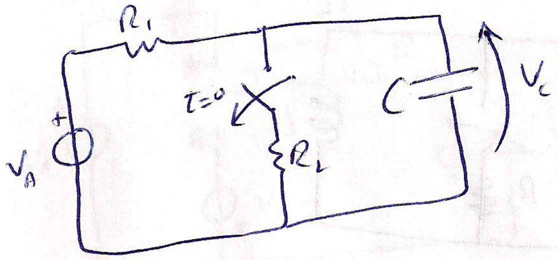
$\boxed{\tau = (R_1 \parallel R_2)\, C}$

d) $V_c(t) = \left(V_c(0) - V_c(\infty)\right) e^{-\frac{t-t_0}{\tau}} + V_c(\infty)$

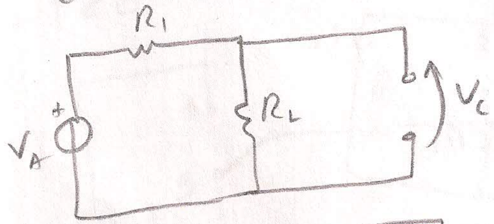
$= \left(V_A - \dfrac{V_A R_2}{R_1+R_2}\right) e^{-t/\tau} + \dfrac{V_A R_2}{R_1+R_2}$

$\boxed{V_c(t) = \dfrac{V_A R_2}{R_1+R_2}\left(1-e^{-t/\tau}\right) + V_A e^{-t/\tau}}$

ii) Closed-to-open

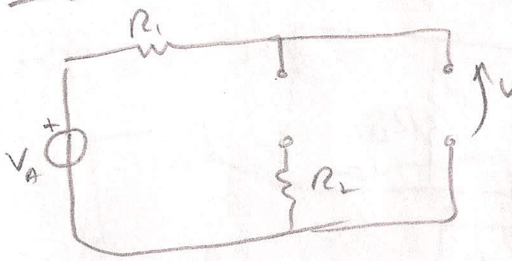


- $t<0$



a) $\boxed{V_c(0) = \dfrac{R_2}{R_1+R_2} V_A}$

- $t \to \infty$



b) $\boxed{V_c(\infty) = V_A}$

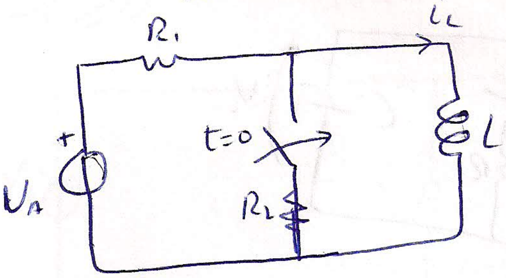
$R_{eq} = R_1$

c) $\boxed{\tau = R_1 \cdot C}$

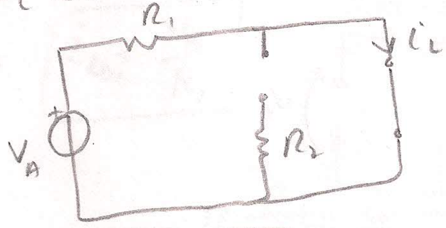
d) $V_c(t) = \left(\dfrac{V_A R_2}{R_1+R_2} - V_A\right) e^{-t/\tau} + V_A$

$\boxed{V_c(t) = \dfrac{V_A R_2}{R_1+R_2} e^{-t/\tau} + V_A\left(1-e^{-t/\tau}\right)}$

9.13 i) Open-To-Closed

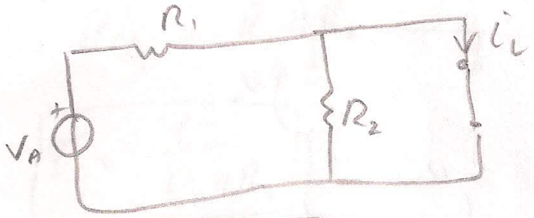


$-t<0$



a) $\boxed{i_L(0) = \frac{V_A}{R_1}}$

$-t \to \infty$



b) $\boxed{i_L(\infty) = \frac{V_A}{R_1}}$

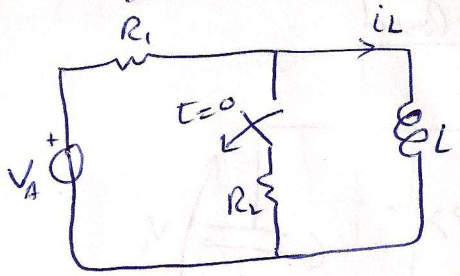
$R_{eq} = R_1 \parallel R_2$

c) $\boxed{\tau = \frac{L}{R_1 \parallel R_2}}$

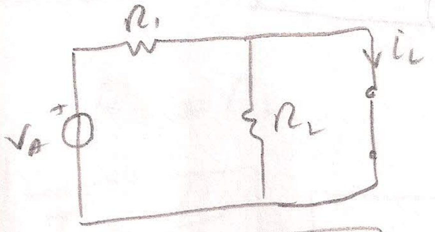
d) $i_L(t) = \left(\frac{V_A}{R_1} - \frac{V_A}{R_1}\right)e^{-t/\tau} + \frac{V_A}{R_1}$

$\boxed{i_L(t) = \frac{V_A}{R_1}}$,

ii) Closed-To-open

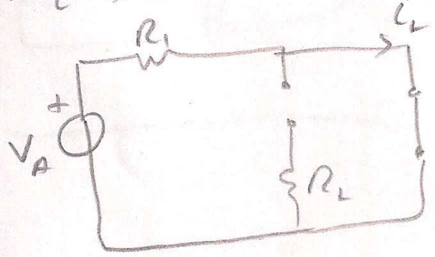


$-t<0$



a) $\boxed{i_L(0) = \frac{V_A}{R_1}}$

$-t \to \infty$



b) $\boxed{i_L(\infty) = \frac{V_A}{R_1}}$

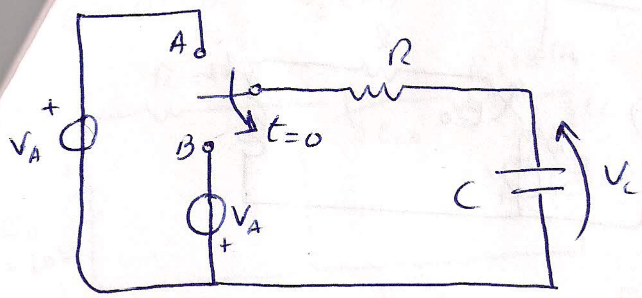
$R_{eq} = R_1$

c) $\boxed{\tau = \frac{L}{R_1}}$

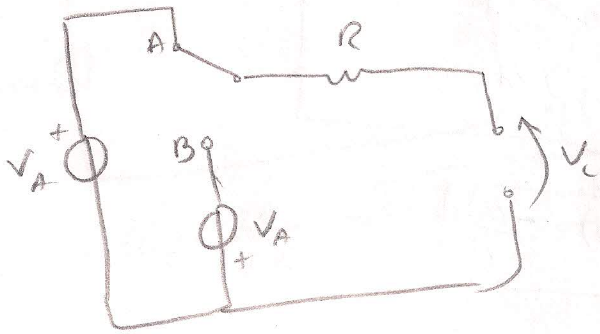
d) $i_L(t) = \left(\frac{V_A}{R_1} - \frac{V_A}{R_1}\right)e^{-t/\tau} + \frac{V_A}{R_1}$

$\boxed{i_L(t) = \frac{V_A}{R_1}}$

i) $A \rightarrow B$

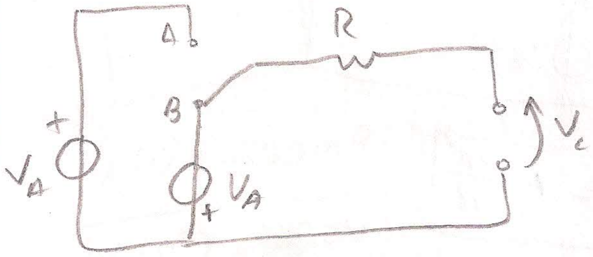


$- t < 0$



$\boxed{V_c(0) = V_A}$

$- t \geq 0$



$\boxed{V_c(\infty) = -V_A}$

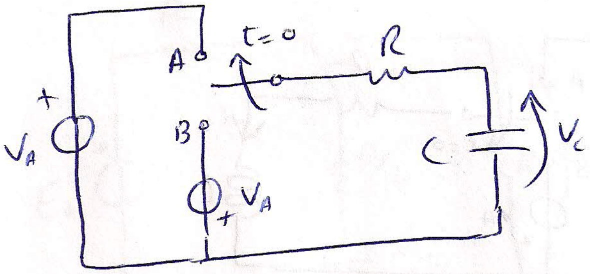
$R_{eq} = R$

$\boxed{\tau = R.C}$

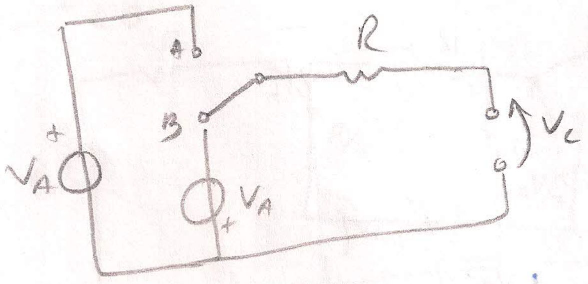
$V_c(t) = (V_A + V_A)e^{-t/\tau} - V_A$

$\boxed{V_c(t) = -V_A + 2V_A e^{-t/\tau}}$

ii) $B \rightarrow A$

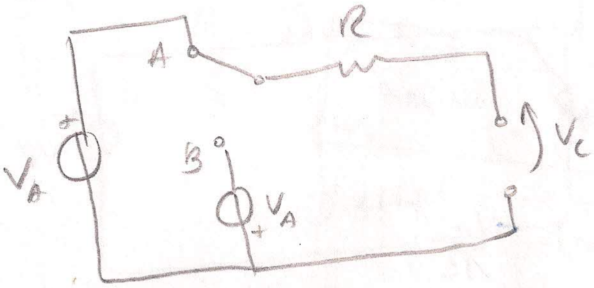


$- t < 0$



$\boxed{V_c(0) = -V_A}$

$- t \geq 0$



$\boxed{V_c(\infty) = V_A}$

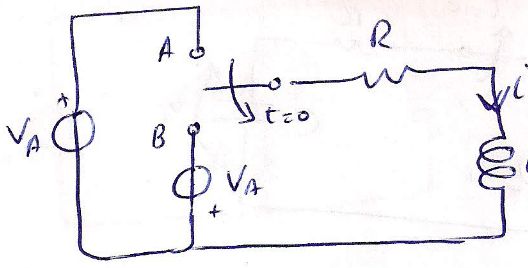
$R_{eq} = R$

$\boxed{\tau = R.C}$

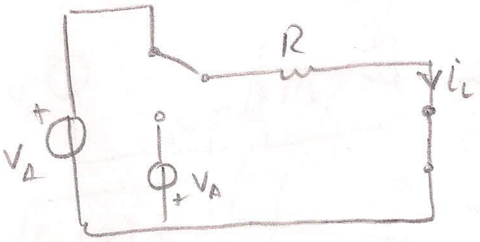
$V_c(t) = (-V_A - V_A)e^{-t/\tau} + V_A$

$\boxed{V_c(t) = V_A - 2V_A e^{-t/\tau}}$

9.15 i) $A \rightarrow B$

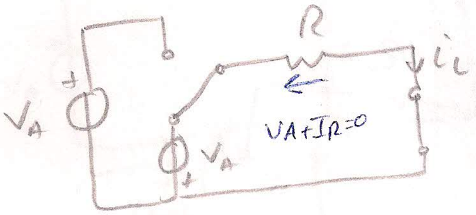


$-t<0$



$$\boxed{i_L(0) = +\frac{V_A}{R}}$$

$-t \geq 0$



$$\boxed{i_L(\infty) = -\frac{V_A}{R}}$$

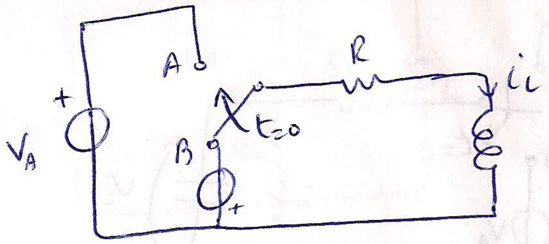
$R_{eq} = R$

$$\boxed{\tau = \frac{L}{R}}$$

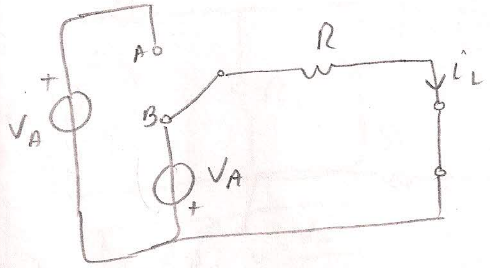
$$i_L(t) = \left(+\frac{V_A}{R} + \frac{V_A}{R}\right) e^{-t/\tau} - \frac{V_A}{R}$$

$$\boxed{i_L(t) = -\frac{V_A}{R} + \frac{2V_A}{R} e^{-t/\tau}}$$

ii) $B \rightarrow A$

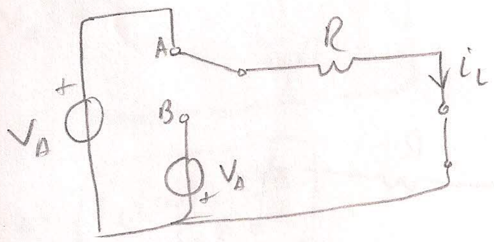


$-t<0$



$$\boxed{i_L(0) = -\frac{V_A}{R}}$$

$-t \geq 0$



$$\boxed{i_L(\infty) = +\frac{V_A}{R}}$$

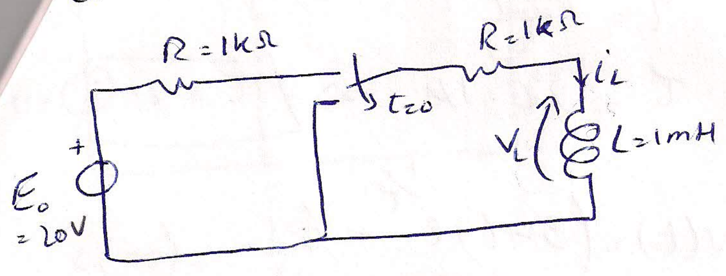
$R_{eq} = R$

$$\boxed{\tau = R.C}$$

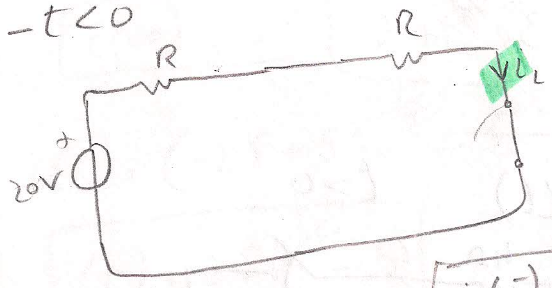
$$i_L(t) = \left(-\frac{V_A}{R} - \frac{V_A}{R}\right) e^{-t/\tau} + \frac{V_A}{R}$$

$$\boxed{i_L(t) = +\frac{V_A}{R} - \frac{2V_A}{R} e^{-t/\tau}}$$

$v_L(0^+)$ & steady state condition $v_L(\infty)$

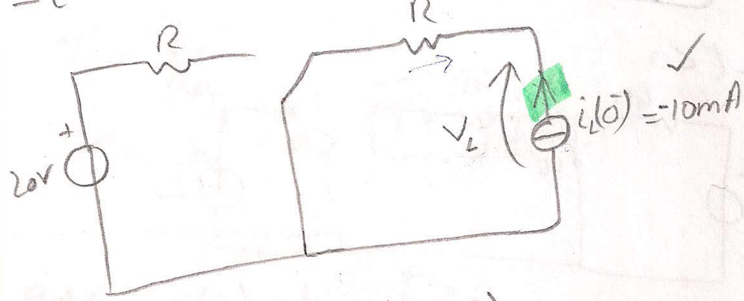


- $t<0$



$i_L(0^-) = \frac{20}{2k} \Rightarrow \boxed{i_L(0^-) = 10mA}$

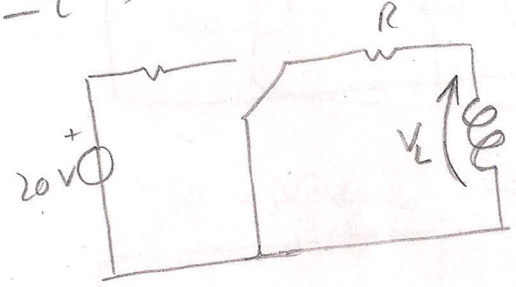
- $t \to 0^+$



$v_L(0^+) = -10mA(1k\Omega)$

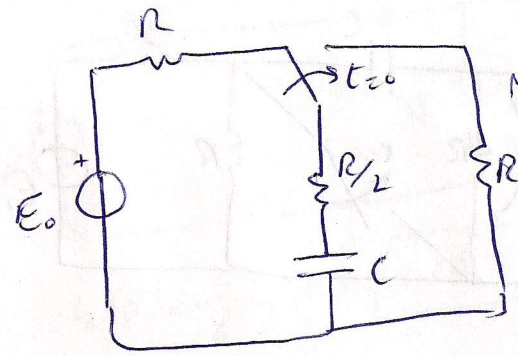
$\boxed{v_L(0^+) = -10V}$

- $t \to +\infty$



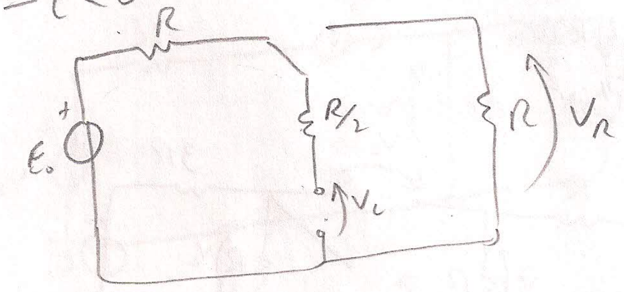
$\boxed{v_L(\infty) = 0}$

9.17 $V_R(0^+) = ?$, $V_R(\infty) = ?$



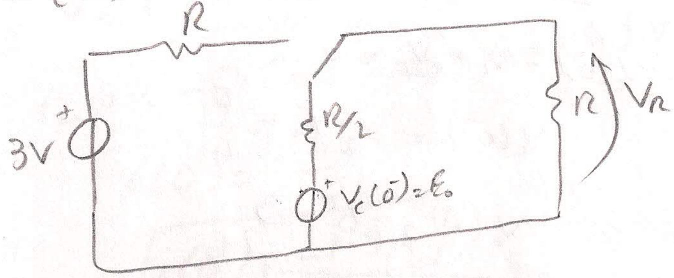
R = 1kΩ
E_0 = 3V
C = 1μF

- $t<0$



$\boxed{V_c(0^-) = E_0}$

- $t \to 0^+$



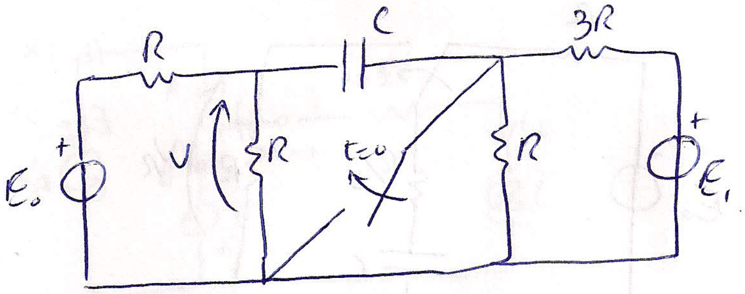
$V_R(0^+) = \frac{R}{\frac{3R}{2}} 3$

$\boxed{V_R(0^+) = 2V}$ ✓

- $t \to \infty$

$\boxed{V_R(\infty) = 0}$

9.18 $v(t)$ $t>0$



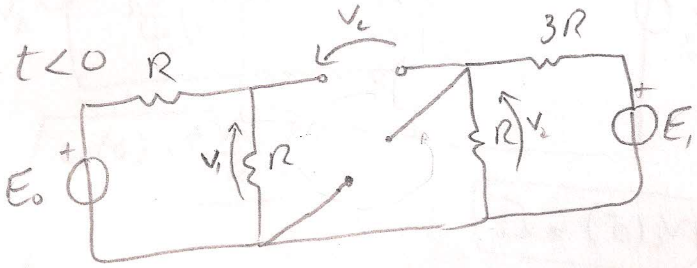
$E_0 = 12V$

$E_1 = 12V$

$R = 4k\Omega$

$C = 1\mu F$

$- t<0$



$V_1 = \frac{R}{2R} E_0$, $V_2 = \frac{R}{4R} E_1$

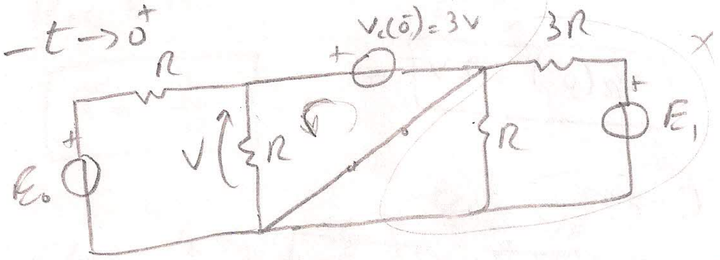
$V_c(0^-) - V_1 + V_2 = 0$

$V_c(0^-) = \frac{1}{2} E_0 - \frac{E_1}{4}$

$V_c(0^-) = 6 - 3$

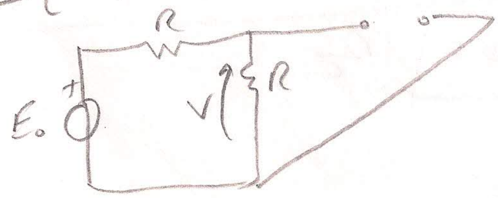
$\boxed{V_c(0^-) = 3V}$

$- t \to 0^+$



$V(0^+) = V_c(0^-) \Rightarrow \boxed{V(0^+) = 3V}$

$- t \to \infty$



$V(\infty) = \frac{E_0}{2} \Rightarrow \boxed{V(\infty) = 6V}$

$R_{eq} = \frac{R}{2} = 2k\Omega$

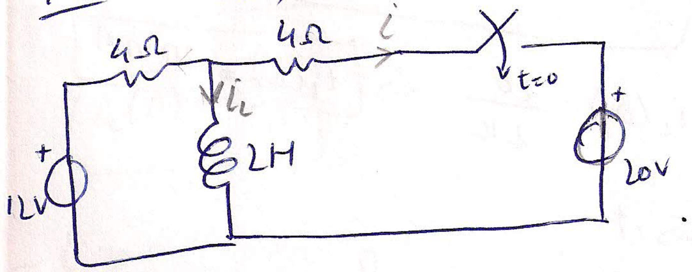
$\tau = R_{eq} C$

$\tau = 2k \cdot 1\mu \Rightarrow \boxed{\tau = 2ms}$

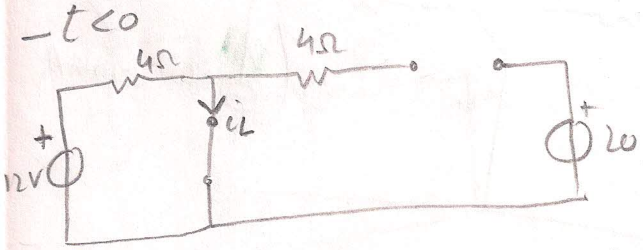
$v(t) = (3-6)e^{-t/\tau} + 6$

$\boxed{v(t) = -3e^{-t/\tau} + 6}$

9.19 $i(t)$ $t>0$

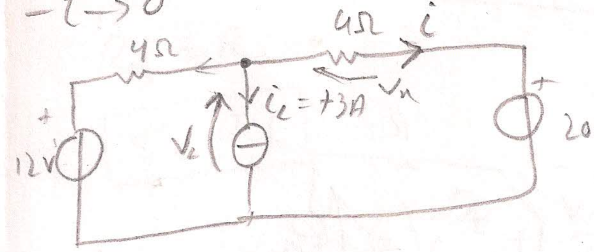


$- t<0$



$i_L(0^-) = +\frac{12}{4} \Rightarrow i_L(0^-) = +3A$

$- t \to 0^+$

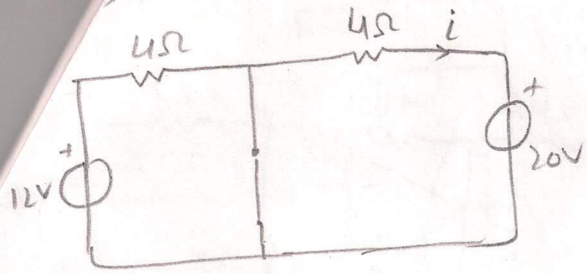


$V_L = \frac{3+5-3}{1/2} \Rightarrow V_L = 10V$

$V_x - V_L + 20 = 0$

$\boxed{V_x = -10V}$ $\boxed{i(0^+) = -2.5A}$

$t \to +\infty$



$i(\infty) = -\frac{20}{4}$

$\boxed{i(\infty) = -5}$

$i(t) = (-2.5 + 5)e^{-t/\tau} - 5$

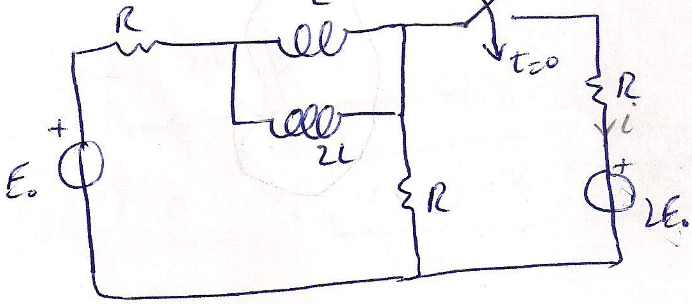
$\boxed{i(t) = 2.5e^{-t/\tau} - 5}$

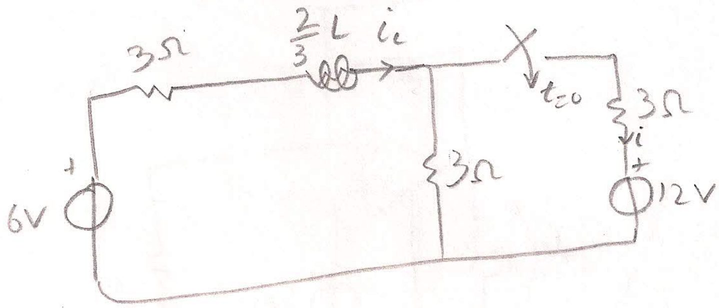
$R_{eq} = \frac{16^2}{8} = 2\Omega$

$\tau = \frac{L}{R_{eq}}$

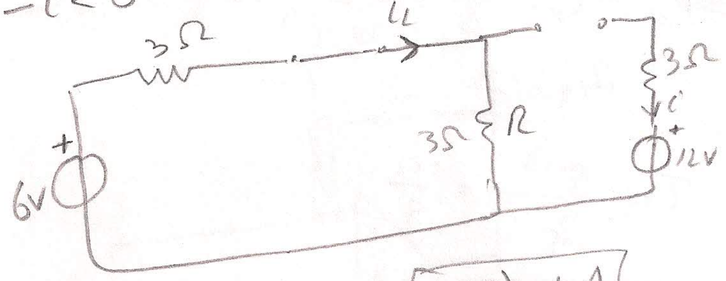
$\tau = \frac{2}{2} \Rightarrow \boxed{\tau = 1s}$

9.20 $i(t) = ?$ $t > 0$



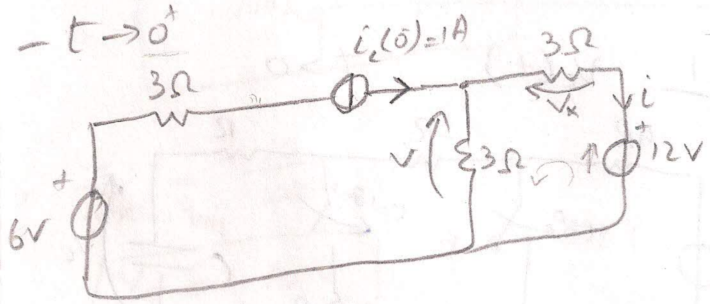


$-t < 0$



$i_L(0^-) = \frac{6}{6} \Rightarrow \boxed{i_L(0^-) = 1A}$

$-t \to 0^+$



$V = \frac{1+4}{\frac{1}{3}+\frac{1}{3}} = \frac{5}{2/3} = \frac{15}{2}$

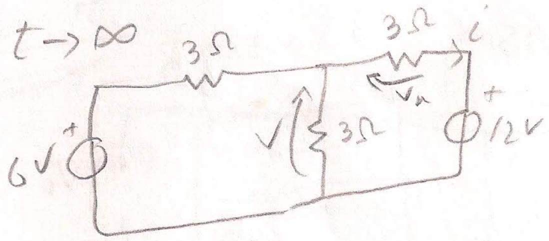
$V = 7.5V$

$12 + V_x - 7.5 = 0 \Rightarrow V_x = -4.5V$

$i(0^+) = \frac{-4.5}{3}$

$\boxed{i(0^+) = -1.5A}$

$-t \to \infty$



$V = \frac{6/3 + 12/3}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \frac{2+4}{1}$

$V = 6V$

$V_x + 12 - 6 = 0 \Rightarrow V_x = -6V$

$i(\infty) = \frac{-6}{3} \Rightarrow \boxed{i(\infty) = -2A}$

$i(t) = (-1.5 + 2)e^{-t/\tau} - 2$

$\boxed{i(t) = 0.5e^{-t/\tau} - 2}$

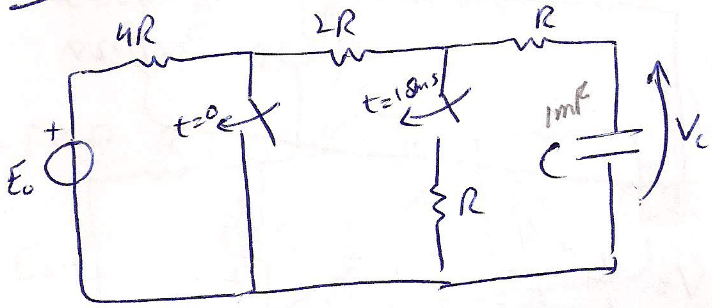
9.21 $R_{eq} = \frac{9}{6} + 3 = \frac{27}{6} = \frac{9}{2}$

$R_{eq} = \frac{9}{2}$

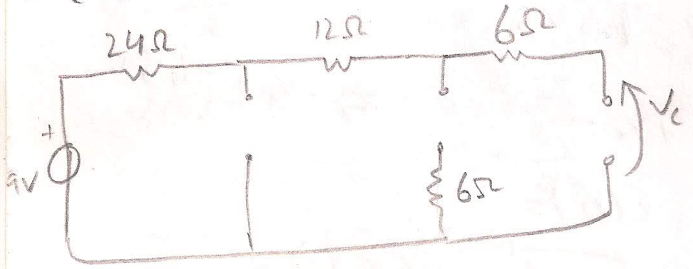
$\tau = \frac{\frac{2}{3}\cdot 3}{\frac{9}{2}} = \frac{4}{9}$

$\boxed{\tau = 0.44 s}$

9.21 $V_c(t)$ $t>0$

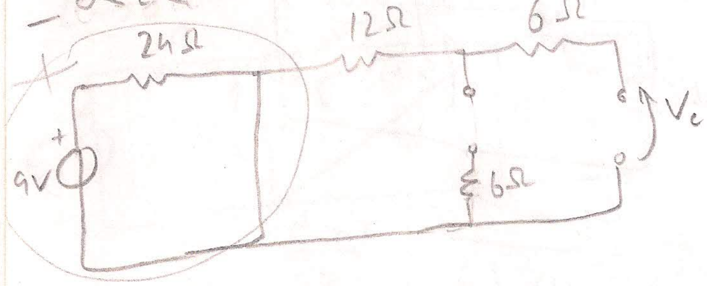


- $t<0$



$\boxed{V_c(0^-) = 9V}$

- $0<t\leq 18ms$



$V_c(\infty) = 0V$

$\boxed{V_c(t) = 9e^{-t/\tau'} V}$

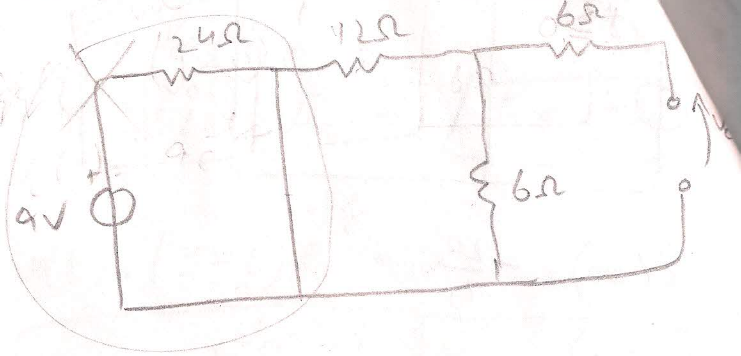
$R_{eq} = 18\Omega$

$\boxed{\tau' = 18ms}$

$V_c(18) = 9e^{-1}$

$\boxed{V_c(18) = 3.31 V}$

- $t>18$



$V_c(\infty) = 0$

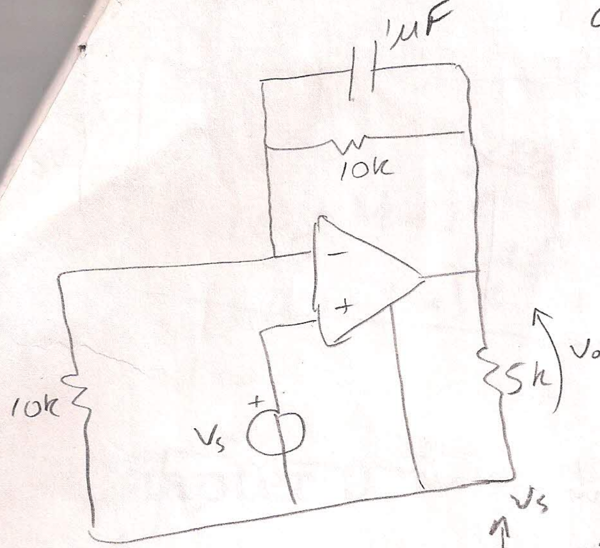
$\boxed{V_c(t) = 3.31\, e^{-\left(\frac{t-18}{\tau''}\right)}}$

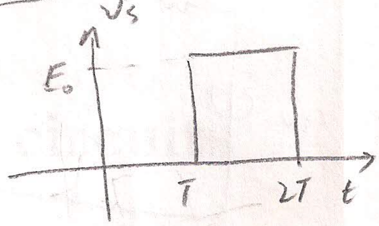
$R_{eq} = \frac{12\cdot 6}{18} + 6$

$R_{eq} = 10\Omega$ ✓

$\boxed{\tau'' = 10ms}$

<u>9.22</u>





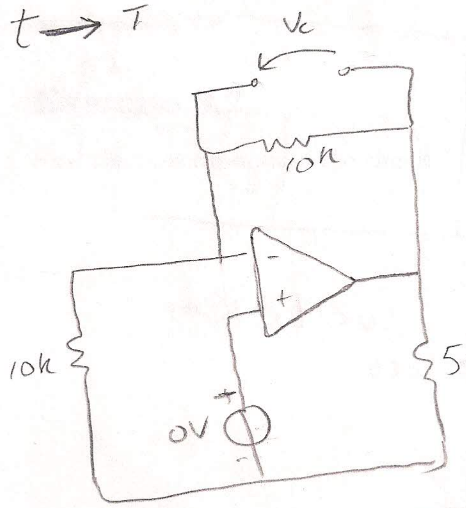
$- t < T$

$V_s = 0$

$V_c(t) = 0$

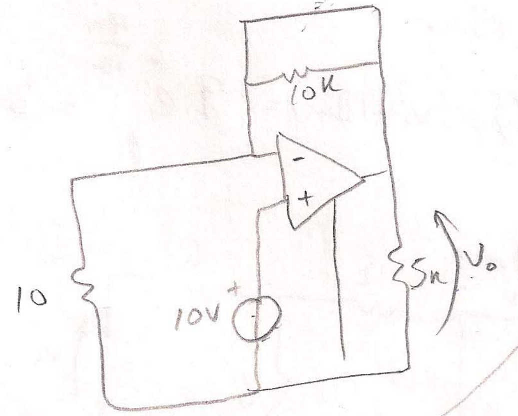
$V_o(t) = 0$

$- t \rightarrow T^-$



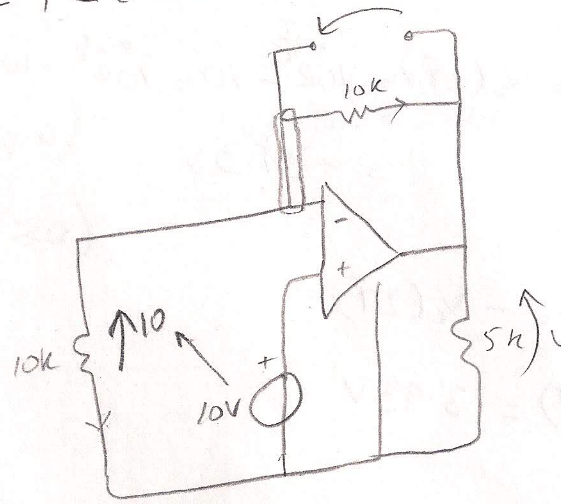
$V_c(T^-) = 0$, $\boxed{V_c(T^+) = V_c(T^-) = 0V}$

$- t \rightarrow T^+$



$\boxed{V_o(T^+) = 10V}$

$- T < t < 2T$



$\frac{V_c}{\cancel{10k}} = -\frac{10}{\cancel{10k}}$

$\boxed{V_c = -10\,V}$ $T < t < 2T$

$V_o + V_c - 10 = 0$

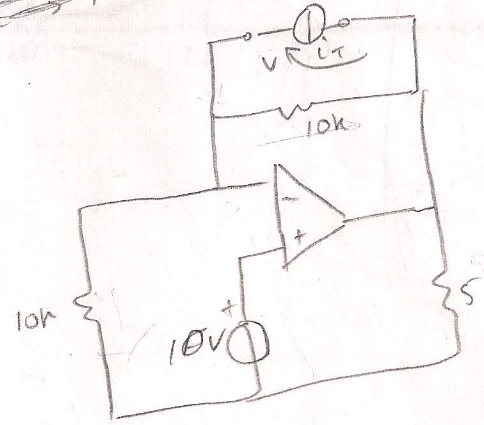
$\boxed{V_o = 20V}$ $T < t < 2T$

$\boxed{V_c(t) = 10e^{-\frac{t-T}{\tau}} - 10\ V}$ $T < t < 2T$

$\boxed{V_o(t) = (10 - 20)e^{-\frac{t-T}{\tau}} + 20\ V}$

<u>For τ</u> $T < t < 2T$

~~t → T~~



$\frac{V}{10k} + \frac{10}{10k} = i_T$

$\frac{V}{10k} = i_T - \frac{10}{10k}$

$V = \underline{10k}\, i_T - 10$

$R_{eq} = 10k$

$\boxed{\tau = 10ms}$

$- t \to 2T^+$

$V_c(2T^+) = V_c(2T^-) = 10e^{-\frac{T}{\tau}} - 10 = 10e^{-\frac{1}{2}} - 10$

$= -3.93\,V$

$V_o(2T^+) = -V_c(2T^+)$

$V_o(2T^+) = 3.93\,V$

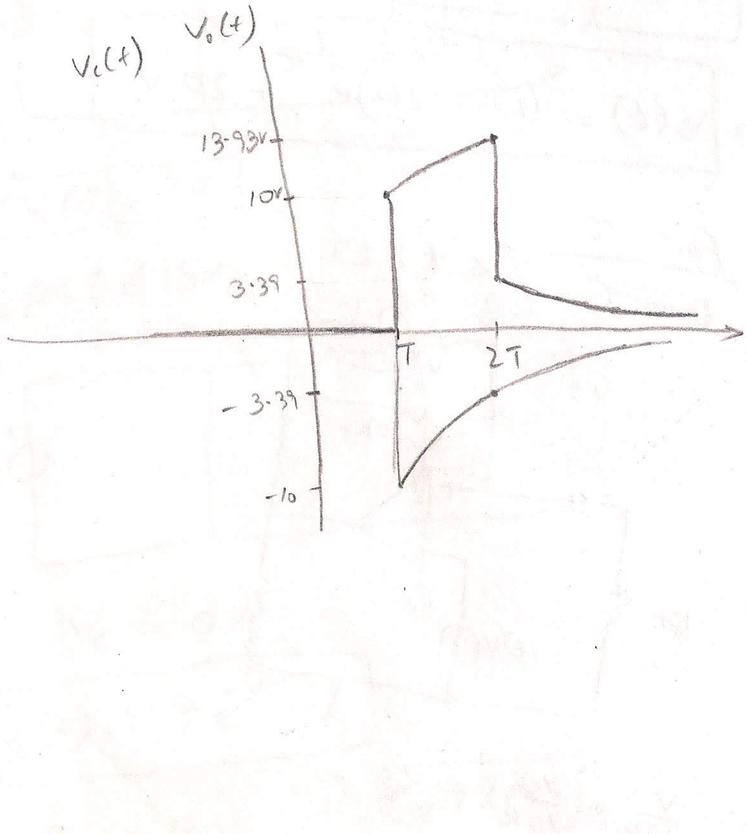
$- t > 2T$

$V_c(\infty) = 0$

$V_o(\infty) = 0$

$$\boxed{\begin{aligned} V_c(t) &= -3.93\, e^{-\frac{t-2T}{\tau}} \\ V_o(t) &= 3.93\, e^{-\frac{t-2T}{\tau}} \end{aligned}}$$

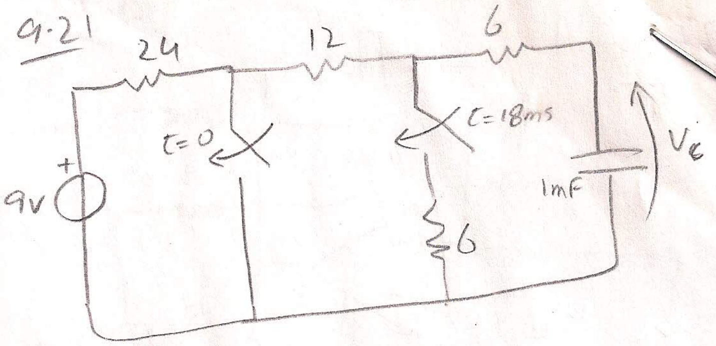
$t > 2T$



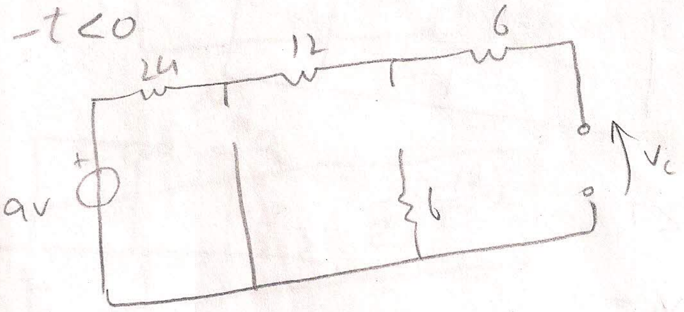
$\tau'' = 10.1m$

$\tau'' = 10ms$

9.21

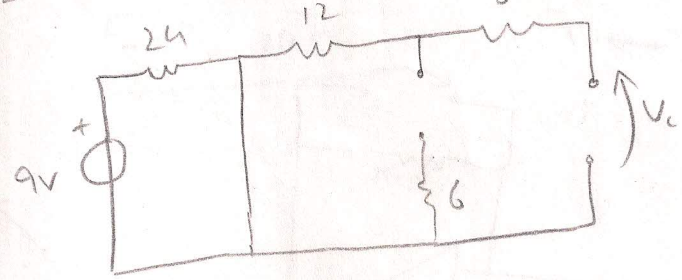


$- t < 0$



$V_c(0^-) = 9V$

$- 0 < t < 18ms$



$V_c = 0 \quad 0 < t < 18ms$

$R_{eq} = 18\,\Omega$

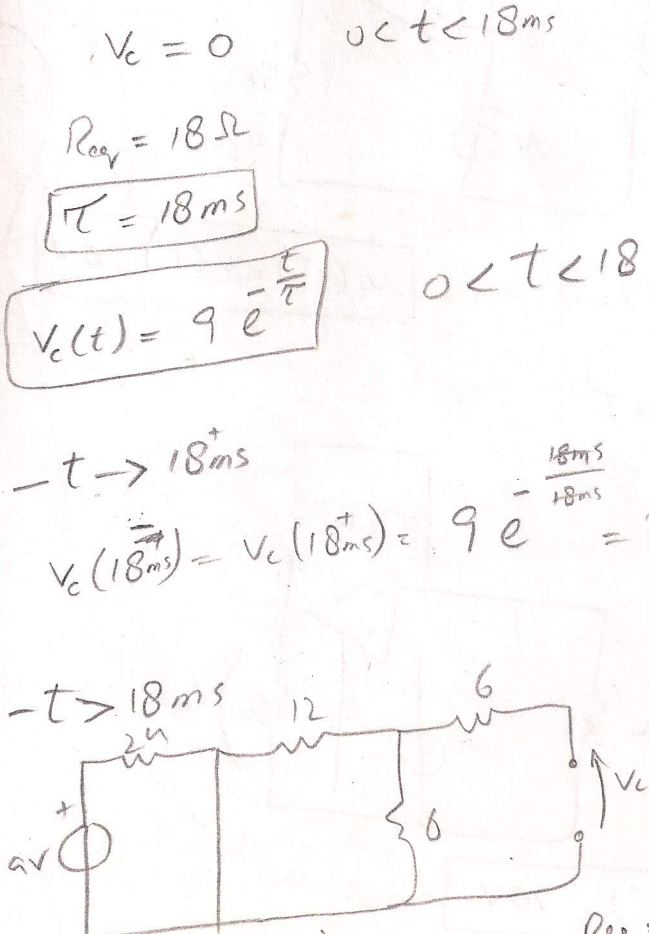
$\boxed{\tau = 18ms}$

$\boxed{V_c(t) = 9e^{-\frac{t}{\tau}}} \quad 0 < t < 18$

$- t \to 18^+ms$

$V_c(18^-ms) = V_c(18^+ms) = 9e^{-\frac{18ms}{18ms}} = 3.31\,V$

$- t > 18ms$



$R_{eq} = 10\,\Omega$

$V_c(\infty) = 0$

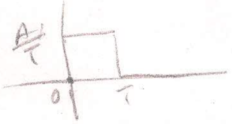
CH#10

BASIC LAPLACE TRANSFORM PAIRS

SIGNAL	WAVEFORM $f(t)$	TRANSFORM $F(s)$
Impulse	$\delta(t)$	1
Step Function	$u(t)$ $\alpha u(\beta t+\varepsilon)$	$\frac{1}{s}$, $\frac{\alpha}{s} e^{\frac{\varepsilon}{\beta}}$
Ramp	$tu(t)$	$\frac{1}{s^2}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s+\alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s+\alpha)^2}$
Sine	$[\sin \beta t]u(t)$	$\frac{\beta}{s^2+\beta^2}$
Cosine	$[\cos \beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped Sine	$[e^{-\alpha t} \sin \beta t]u(t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
Damped Cosine	$[e^{-\alpha t} \cos \beta t]u(t)$	$\frac{(s+\alpha)}{(s+\alpha)^2+\beta^2}$
Simple Complex Poles	$[2\|k\|e^{-\alpha t} \cos(\beta t + \angle k)]u(t)$	$\frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta}$
Double Complex Poles	$[2\|k\|te^{-\alpha t} \cos(\beta t + \angle k)]u(t)$	$\frac{k}{(s+\alpha-j\beta)^2} + \frac{k^*}{(s+\alpha+j\beta)^2}$

BASIC LAPLACE TRANSFORMATION PROPERTIES

Properties	Time Domain	Frequency Domain
Independent Variable	t	s
Signal Representation	$f(t)$	$F(s)$
Uniqueness	$\mathcal{L}^{-1}\{F(s)\}(=)[f(t)]u(t)$	$\mathcal{L}\{f(t)\} = F(s)$
Linearity	$Af_1(t) + Bf_2(t)$	$AF_1(s) + BF_2(s)$
Integration	$\int_0^t f(\tau)d\tau$	$\dfrac{F(s)}{s}$
Differentiation	$\dfrac{df(t)}{dt}$	$sF(s) - f(0-)$
	$\dfrac{d^2 f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f'(0-)$
	$\dfrac{d^3 f(t)}{dt^3}$	$s^3F(s) - s^2 f(0-) - sf'(0-) - f''(0-)$
t-Translation	$[f(t-a)]u(t-a)$	$e^{-as}F(s)$
s-Translation	$e^{-\alpha t}f(t)$	$F(s+\alpha)$
Scaling	$f(at)$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$
Final Value	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
Initial Value	$\lim_{t\to 0+} f(t)$	$\lim_{s\to\infty} sF(s)$

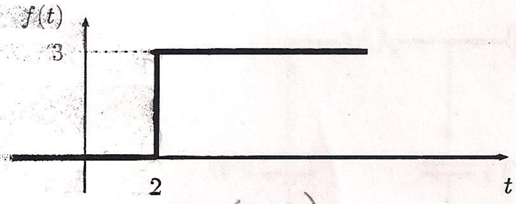


$\frac{A}{T}u(t) - \frac{A}{T}u(t-T)$

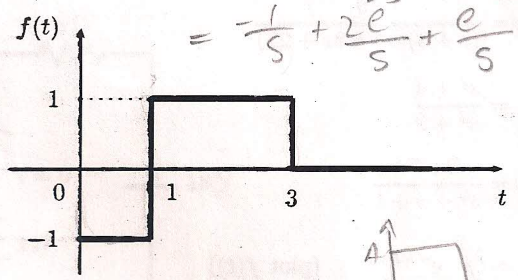
$-u(t)+2u(t-1)+u(t-3)$

$= -\frac{1}{s} + \frac{2e^{-s}}{s} + \frac{e^{-3s}}{s}$

1)



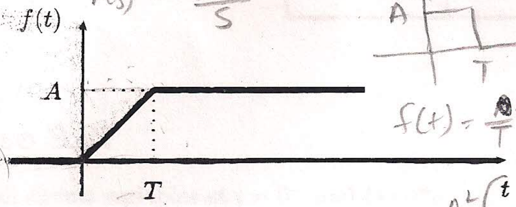
2)



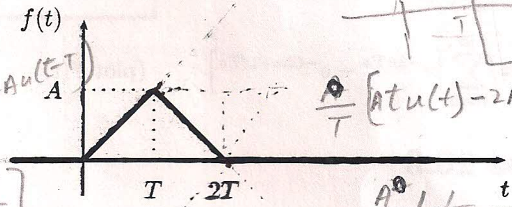
$f(t) = 3u(t-2)$

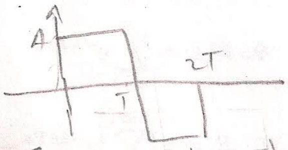
$F(s) = \frac{3e^{-2s}}{s}$

3)



4)





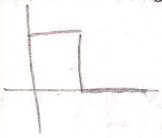
$f(t) = \frac{A}{T}t\left[Au(t) - Au(t-T)\right]$

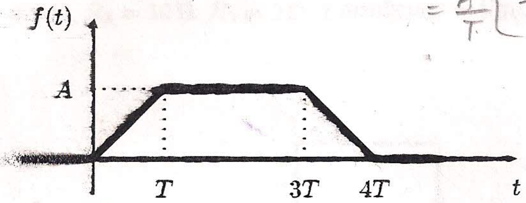
$= \frac{A^2}{T}\left(\frac{1}{s^2} - \frac{e^{-Ts}}{s^2}\right)$

$\frac{A}{T}\left[Atu(t) - 2Atu(t-T) + Atu(t-2T)\right]$

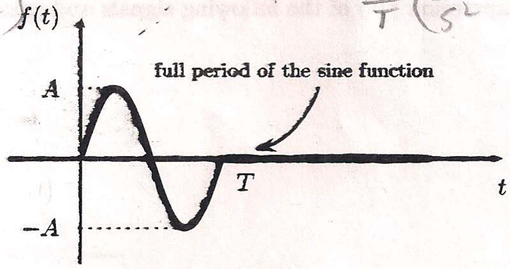
$\frac{A}{T}\left(\frac{1}{s^2} - \frac{2e^{-Ts}}{s^2} + \frac{e^{-2Ts}}{s^2}\right)$

5)

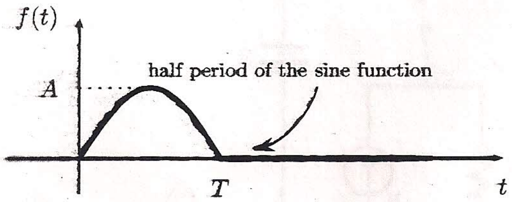




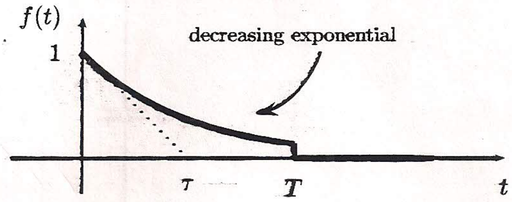
6)



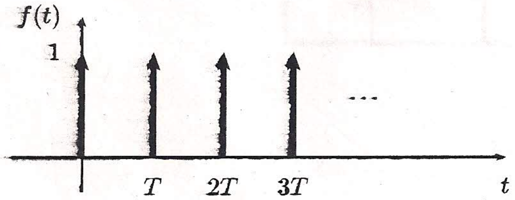
7)



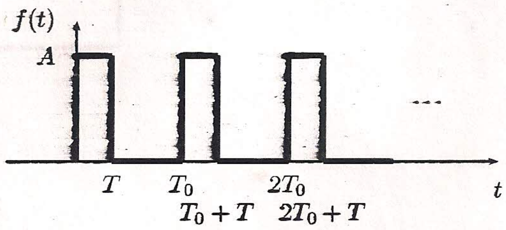
8)



9)



10)



S221919@studenti.polito.it

i) $f(t) = 3u(t-2)$

ii) $f(t) = -u(t) + 2u(t-1) - u(t-3)$

iii) $f(t) = \frac{A}{T}\delta(t) - \frac{A}{T}\delta(t-T)$

iv) $f(t) = \frac{A}{T}\delta(t) - \frac{2A}{T}\delta(t-T) + \frac{A}{T}\delta(t-2T)$

v) $f(t) = \frac{A}{T}\delta(t) - \frac{A}{T}\delta(t-T) - \frac{A}{T}\delta(t-3T) + \frac{A}{T}\delta(t-4T)$

lculate Laplace transform.

1) $f(t) = e^{-at} - e^{-bt}$

$$F(s) = \frac{1}{s+a} - \frac{1}{s+b}$$

2) $f(t) = \sin(\omega t + \theta)$

$$F(s) = \mathcal{L}\{f(t)\}$$

$$= \frac{e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}}{2j}$$

$$= \frac{e^{j\theta}}{2j}\cdot e^{j\omega t} - \frac{e^{-j\theta}}{2j} e^{-j\omega t}$$

$$= \frac{e^{j\theta}}{2j}\cdot\frac{1}{s-j\omega} - \frac{e^{-j\theta}}{2j}\cdot\frac{1}{s+j\omega}$$

$$= \frac{e^{j\theta}(s+j\omega) - e^{-j\theta}(s-j\omega)}{2j(s-j\omega)(s+j\omega)}$$

$$= \frac{\left(\frac{e^{j\theta}-e^{-j\theta}}{2j}\right)s + \frac{e^{j\theta}+e^{-j\theta}}{2\cancel{j}}\cancel{j}\omega}{s^2+\omega^2}$$

$$= \frac{\sin\theta\, s + \cos\theta\,\omega}{s^2+\omega^2}$$

$$F(s) = \frac{s.\sin\theta + \omega\cos\theta}{s^2+\omega^2} \quad A$$

$\sin\omega t\cos\theta + \cos\omega t\sin\theta$

$\frac{\cos\theta\,\omega}{s^2+\omega^2} + \frac{\sin\theta\, s}{s^2+\omega^2}$

$e^{-4t+12-12}u(t-3)$

$e^{-12} e^{-4(t-3)} u(t-3)$ $\quad e^{-12}e^{-3s}\frac{1}{s+4}$ $\left(\frac{e^{-3(s+4)}}{s+4}\right)$

$= e^{-12}e^{-3s}\frac{1}{s+4} = e^{-3(s+4)}\frac{1}{s+4}$

3) $f(t) = e^{-\alpha t}\cos(\omega t+\theta)$

$$F(s) = \frac{e^{-\alpha t}e^{j(\omega t+\theta)} + e^{-\alpha t}e^{-j(\omega t+\theta)}}{2}$$

$$= \frac{e^{j\theta}}{2}\cdot e^{(-\alpha+j\omega)t} + \frac{e^{-j\theta}}{2}e^{(-\alpha-j\omega)t}$$

$$= \frac{e^{j\theta}}{2}\cdot\frac{1}{s+\alpha-j\omega} + \frac{e^{-j\theta}}{2}\cdot\frac{1}{s+\alpha+j\omega}$$

$$= \frac{e^{j\theta}(s+\alpha+j\omega)+e^{-j\theta}(s+\alpha-j\omega)}{2(s+\alpha-j\omega)(s+\alpha+j\omega)}$$

$$= \frac{\left(\frac{e^{j\theta}+e^{-j\theta}}{2}\right)(s+\alpha) + \left(\frac{e^{j\theta}-e^{-j\theta}}{2j}\right)j^2\omega}{(s+\alpha)^2+\omega^2}$$

$$= \frac{\cos\theta\,(s+\alpha)+\sin\theta\, j^2\omega}{(s+\alpha)^2+\omega^2}$$

$$= \frac{(s+\alpha)\cos\theta - \omega\sin\theta}{(s+\alpha)^2+\omega^2}$$

4) $f(t) = e^{-2t} + \sin t$

$$F(s) = \frac{1}{s+2} + \frac{1}{s^2+1}$$

5) $f(t) = e^{-4t}u(t-3)$

$$f_1(t) = u(t-3) \xrightarrow{\mathcal{L}} F_1(s) = \frac{e^{-3s}}{s}$$

$$f(t) = e^{-4t}u(t-3) \xrightarrow{\mathcal{L}} F(s) = F_1(s+4)$$

$$F(s) = \frac{e^{-3(s+4)}}{s+4}$$

6) $f(t) = \sin(t-\tau)\,u(t-\tau)$

$$= F(s)\,e^{-\tau s} = \mathcal{L}\{\sin t\,u(t)\}\,e^{-\tau s}$$

$$F(s) = \frac{1}{s^2+1}\,e^{-\tau s}$$

---

7) $f(t) = \delta(t) + 2u(t) - 3e^{-2t}$

$$= 1 + \frac{2}{s} - \frac{3}{s+2}$$

---

8) $f(t) = \delta(t) - \delta(t-T)$

$$= 1 - \mathcal{L}\{\delta(t)\,u(t)\}\,e^{-Ts}$$

$$= 1 - e^{-Ts}$$

---

9) $f(t) = \sum_{n=0}^{\infty} \delta(t-nT)$

$$= \sum_{n=0}^{\infty} \mathcal{L}\{\delta(t)\}\,e^{-nTs}$$

$$= \sum_{n=0}^{\infty} e^{-nTs}$$

let $|\alpha| = e^{-Ts}$

$$= \sum_{n=0}^{\infty} |\alpha|^n$$

if $|\alpha| < 1$

$$= \frac{1}{1-|\alpha|}$$

$$= \frac{1}{1-e^{-sT}} \text{ Ans}$$

10) $f(t) = \sum_{n=0}^{\infty} (-1)^n u(t-nT)$

$$= \sum_{n=0}^{\infty} (-1)^n \mathcal{L}\{u(t)\}\,e^{-sTn}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{s} e^{-sTn}$$

$$= \sum_{k=0}^{\infty} \frac{1}{s}\left(e^{-sT}\right)^{2k} - \sum_{k=0}^{\infty} \frac{1}{s}\left(e^{-sT}\right)^{2k+1}$$

$$= \sum_{k=0}^{\infty} \left(e^{-sT}\right)^{2k}\left[\frac{1}{s} - \frac{1}{s}e^{-sT}\right]$$

$$= \frac{1}{s}\left(1-e^{-sT}\right)\sum_{k=0}^{\infty} e^{-2skT}$$

$$= \frac{1}{s}\left(1-e^{-sT}\right)\cdot\frac{1}{1-e^{-2sT}}$$

$$= \frac{1}{s}\cdot\frac{1-e^{-sT}}{1-e^{-2sT}} \text{ Ans}$$

---

11) $f(t) = u(4t)$

$$= \frac{1}{s}$$

---

12) $f(t) = u(4t-3)$

$$= u\left[4\left(t-\tfrac{3}{4}\right)\right]$$

$$= \frac{1}{s}e^{-\frac{3}{4}s} \text{ Ans}$$

.2

Calculate inverse laplace

1) $F(s) = \frac{3}{s} - \frac{5}{s+1}$

$= 3u(t) - 5e^{-t}u(t)$

$f(t) = (3 - 5e^{-t})u(t)$

---

2) $F(s) = \frac{6}{s^2+4}$

$= 3 \cdot \frac{2}{(s)^2 + 2^2}$

$= 3\sin(2t)\,u(t)$

---

3) $F(s) = \frac{s^2+12}{s(s+2)(s+3)}$

$= \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+3} = F(s)$

$A_1 = \lim_{s\to 0} s\,F(s) = \frac{s^2+12}{(s+2)(s+3)}\Big|_0 = 2$

$A_2 = \lim_{s\to -2} (s+2)F(s) = \frac{s^2+12}{s(s+3)}\Big|_{-2} = \frac{16}{-2} = -8$

$A_3 = \lim_{s\to -3} (s+3)F(s) = \frac{s^2+12}{s(s+2)}\Big|_{-3} = \frac{21}{3} = 7$

$F(s) = \frac{2}{s} + \frac{-8}{s+2} + \frac{7}{s+3}$

$f(t) = (2 - 8e^{-2t} + 7e^{-3t})u(t)$ Ans

4) $F(s) = \frac{6(s+2)}{(s+1)(s+3)(s+4)}$

$F(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3} + \frac{A_3}{s+4}$

$A_1 = \lim_{s\to -1} (s+1)F(s) = \frac{6(s+2)}{(s+3)(s+4)}\Big|_{-1} = \frac{6}{6} = 1$

$A_2 = \lim_{s\to -3} (s+3)F(s) = \frac{6(s+2)}{(s+1)(s+4)}\Big|_{-3} = \frac{-6}{-2} = 3$

$A_3 = \lim_{s\to -4} (s+4)F(s) = \frac{6(s+2)}{(s+1)(s+3)}\Big|_{-4} = \frac{-12}{3} = -4$

$F(s) = \frac{1}{s+1} + \frac{3}{s+3} + \frac{-4}{s+4}$

$f(t) = (e^{-t} + 3e^{-3t} + -4e^{-4t})u(t)$ Ans

---

5) $F(s) = \frac{10s^2+4}{s(s+1)(s+2)^2}$

$F(s) = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{R_1}{(s+2)^2} + \frac{R_2}{s+2}$

$A_1 = \frac{10s^2+4}{(s+1)(s+2)^2}\Big|_0 = \frac{4}{4} = 1$

$A_2 = \frac{10s^2+4}{s(s+2)^2}\Big|_{-1} = \frac{14}{-1} = -14$

$R_1 = \frac{10s^2+4}{s(s+1)}\Big|_{-2} = \frac{44}{2} = 22$

$R_2 = \lim_{s\to -2} \frac{d}{ds}\left(\frac{10s^2+4}{s(s+1)}\right) = \lim_{s\to -2} \frac{s(s+1)20s - (10s^2+4)\frac{d}{ds}s(s+1)}{s^2(s+1)^2}$

$= \lim_{s\to -2} \frac{20s^2(s+1) - (10s^2+4)[s+s+1]}{s^2(s+1)^2}$

$= \frac{-80 - 44(-3)}{4} = \frac{-80+132}{4} = \frac{52}{4}$

$= 13$

[redacted] AKHTAR

$$F(s) = \frac{1}{s} + \frac{-14}{s+1} + \frac{22}{(s+2)^2} + \frac{13}{s+2}$$

$$f(t) = \left(1 - 14e^{-t} + 22te^{-2t} + 13e^{-2t}\right) u(t)$$

6) $F(s) = \dfrac{10}{(s+3)(s^2+8s+25)}$

$s^2+8s+25$

$$P = \frac{-8 \pm \sqrt{64-100}}{2}$$

$$= \frac{-8 \pm 6i}{2}$$

$$= -4 \pm 3i$$

$P = -4+3i \qquad P^* = -4-3i$

$$\frac{10}{(s+3)(s^2+8s+25)} = \frac{A}{s+3} + \frac{B}{s-p} + \frac{B^*}{s-p^*}$$

$$A = \lim_{s \to -3} \frac{10}{s^2+8s+25} = \frac{10}{9-24+25} = 1$$

$$B = \lim_{s \to p} \frac{10}{(s+3)(s-p^*)}$$

$$= \frac{10}{(-4+3i+3)(-4+3i+4+3i)}$$

$$= \frac{\cancel{10}\,5}{(-1+3i)(\cancel{6}\,3\,i)} = \frac{5}{-3i-9}$$

$$= \frac{5}{-3i-9} \times \frac{-9+3i}{-9+3i} = \frac{\cancel{15}(\cancel{3}+i)}{\cancel{81+9}\;6}$$

$$B = \frac{-3+i}{6} \Rightarrow B' = \frac{-3-i}{6}$$

$$\frac{B}{s-p} + \frac{B^*}{s-p^*} = \frac{-\frac{1}{2} + \frac{i}{6}}{s+4-3i} + \frac{-\frac{1}{2} - \frac{i}{6}}{s+4+3i}$$

$$= \frac{2(-\frac{1}{2})(s+4) + 2(\frac{i}{6})(3i)}{(s+4)^2+9}$$

$$= \frac{-s-4-1}{(s+4)^2+9}$$

$$= \frac{-s-5}{(s+4)^2+9}$$

$$F(s) = \frac{1}{s+3} + \frac{-s-5}{(s+4)^2+9}$$

$$= e^{-3t} - \frac{s+4}{(s+4)^2+9} - \frac{3}{(s+4)^2+9} \times \frac{1}{3}$$

$$= \left(e^{-3t} - e^{-4t}\cos(3t) - \frac{1}{3}e^{-4t}\sin(3t)\right) u(t)$$

$$\frac{s^2+4}{s^2+9} = F(s)$$

$$s^2+9 \overline{\big)\, s^2+4} \quad 1$$
$$\frac{s^2+9}{-5}$$

$$F(s) = 1 - \frac{5}{s^2+9}$$

$$s = \pm 3$$

$$\frac{5}{s^2+9} = \frac{A}{s-3i} + \frac{B}{s+3i}$$

$$A = \lim_{s\to 3i} \frac{5}{s+3i} = \frac{5}{3i+3i}$$

$$= \frac{5}{6i} \times \frac{6i}{6i} = \frac{\cancel{30i}\,5}{\cancel{-36}\,6}$$

$$A = -\frac{5}{6}i$$

$$B = \lim_{s\to -3i} \frac{5}{s-3i} = \frac{5}{-3i-3i}$$

$$= -\frac{5}{6i} \times \frac{6i}{6i} = \frac{5}{6}i$$

$$F(s) = 1 - \left(\frac{\frac{-5}{6}i}{s-3i} + \frac{\frac{5}{6}i}{s+3i} \right)$$

$$= 1 + \frac{5}{6}i\left(\frac{1}{s-3i}\right) - \frac{5}{6}i\,\frac{1}{s+3i}$$

$$= 1 + \frac{5}{6}i\left[\frac{\cancel{s}+3i-\cancel{s}+3i}{s^2+9}\right]$$

$$= 1 + \frac{5i}{6}\left(\frac{6i}{s^2+9}\right)$$

$$F(s) = 1 - \frac{5}{s^2+9}$$

$$= 1 - \frac{5}{3}\,\frac{3}{s^2+9}$$

$$f(t) = \delta(t) - \frac{5}{3}\sin(3t)\,u(t)$$

---

8) $F(s) = \dfrac{5e^{-6s}}{s^2+s+1}$

$$s^2+s+1 = 0$$

$$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm 3i}{2}$$

$$P = -\frac{1}{2} + \frac{3}{2}i, \quad P^* = -\frac{1}{2} - \frac{3}{2}i$$

$$\frac{5e^{-6s}}{s^2+s+1} = \frac{A}{s-P} + \frac{A^*}{s-P^*}$$

$$A = \lim_{s\to P} \frac{5e^{-6s}}{s+\frac{1}{2}+\frac{3}{2}i}$$

$$= \frac{5e^{3-9i}}{-\frac{1}{2}+\frac{3}{2}i+\frac{1}{2}+\frac{3}{2}i}$$

$$= \frac{5e^{3(1-3i)}}{3i} \times \frac{3i}{3i}$$

$$= \frac{\cancel{15}\,5\,e^{3(1-3i)}\,i}{\cancel{-9}\,3} = -\frac{5}{3}e^{3(1-3i)}\,i$$

$$A^* = \lim_{s\to P^*} \frac{5e^{-6s}}{s+\frac{1}{2}-\frac{3}{2}i}$$

$F(s)$ $s+6$ $G(s)$

$$A^* = \frac{5e^{3(1+3i)}}{-\frac{1}{2}-\frac{3}{2}i+\cancel{\frac{1}{2}}-\frac{3}{2}i}$$

$$= \frac{5e^{3(1+3i)}}{-3i} \times \frac{3i}{3i}$$

$$= \frac{5}{3}e^{3(1+3i)}\, i$$

$$F(s) = \frac{-\frac{5}{3}e^{3(1-3i)}\, i}{s+\frac{1}{2}-\frac{3}{2}i} + \frac{\frac{5}{3}e^{3(1+3i)}\, i}{s+\frac{1}{2}+\frac{3}{2}i}$$

$$= \frac{5}{3}i\left[\frac{-e^{3(1-3i)}}{(s+\frac{1}{2})^2+\frac{9}{4}}\right.$$

$$G(s) = \frac{+\frac{5}{3}i}{s+\frac{1}{2}-\frac{3}{2}i} + \frac{\frac{5}{3}i}{s+\frac{1}{2}+\frac{3}{2}i}$$

$$= \frac{2(-\frac{5}{3}i)(\frac{3}{2}i)}{(s+\frac{1}{2})^2+\frac{9}{4}}$$

$$= \frac{5}{(s+\frac{1}{2})^2+\frac{9}{4}}$$

$$= \frac{5}{3/2}\cdot\frac{3/2}{(s+\frac{1}{2})^2+\frac{9}{4}}$$

$$= e^{-\frac{1}{2}t}\,\frac{10}{3}\sin\left(\frac{3}{2}t\right)u(t)$$

$$e^{-6s}G(s) = \frac{10}{3}e^{-\frac{1}{2}(t-6)}\sin\left(\frac{3}{2}(t-6)\right)u(t-6)$$

wrong

9) $F(s) = \sum_{n=0}^{\infty} e^{-nTs}$

$$G(s) = \sum_{n=0}^{\infty} 1$$

$$g(t) = \delta(t)$$

$$e^{-nTs}G(s) = \delta(t-nT)$$

10) $F(s) = \sum_{n=0}^{\infty}\left[e^{-2nTs} - e^{-(2n+1)Ts}\right]$

$$F(s) = (-1)^n \sum_{n=0}^{\infty} e^{-nTs}$$

$$f(t) = \sum_{n=0}^{\infty}(-1)^n\,\delta(t-nT)$$

$$\sum_0^{\infty}(-n)$$

$$\sum_0^{\infty}(2n-(2n+1))$$

$$\sum_0^{\infty}(-2)^n$$

$$\sum(2^{2n}-2^{2n+1})$$

$$= \frac{5}{(s+\frac{1}{2})^2+\frac{3}{4}}\cdot e^{-6s}$$

$$= \frac{2}{\sqrt{3}}\,\frac{5\cdot\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\,e^{-6s}$$

$$= \frac{10}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}(t-6)\right)e^{-\frac{1}{2}(t-6)}\,u(t-6)$$

$\cancel{F(s) = \frac{1}{(s+1)(s+3)^3}}$ $\cancel{\Rightarrow f(t) = ?}$

$$F(s) = \frac{1}{(s+1)(s+3)^3}$$

$$F(s) = \frac{A}{s+1} + \frac{B_1}{s+3} + \frac{B_2}{(s+3)^2} + \frac{B_3}{(s+3)^3}$$

$$A = \lim_{s \to -1} \frac{1}{(s+3)^3} = \frac{1}{8}$$

$$B_3 = \lim_{s \to -3} \frac{1}{s+1} = -\frac{1}{2}$$

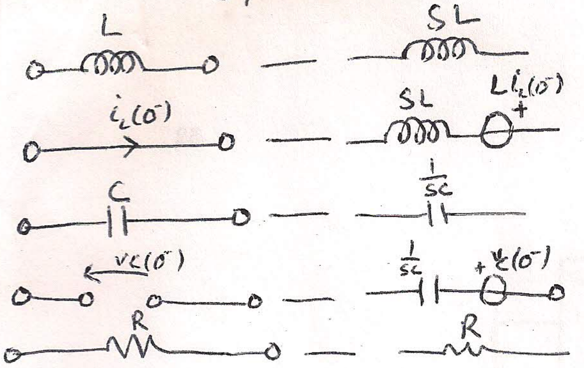
$$B_2 = \lim_{s \to -3} \frac{d}{ds}\left(\frac{1}{s+1}\right) = --$$

$$B_1 = \lim_{s \to -3} \frac{1}{2!} \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)$$

$$(s+3)^3 F(s) = G(s) = G(-3) + G'(-3)(s+3) + \boxed{\frac{G''(-3)}{2!}}(s+3)^2$$

BY PROFFESSOR

SYMBOLIC CKT:



If $V_c(0^-)$, ∀ constant values $\rightarrow \frac{V_c(0^-)}{S}$

$3V \rightarrow \frac{3}{S}$

If $i_L(0^-)$, ∀ constant values $\rightarrow \frac{i_L(0^-)}{S}$

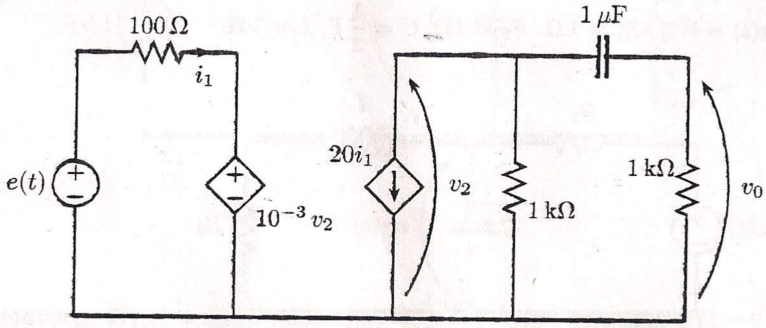
$4A \rightarrow \frac{4}{S}$

Chapter 11

Laplace trasform II: application to circuits

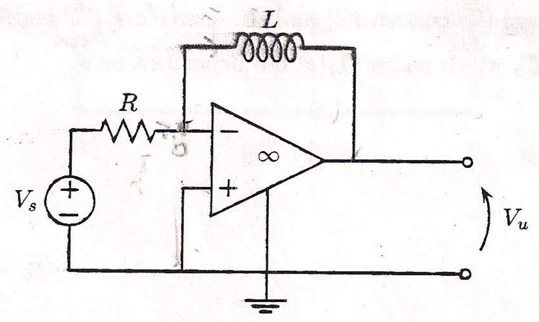
Exercise 11.1

Calculate $H(s) = \frac{V_0(s)}{E(s)}$ and $h(t)$.



Exercise 11.2

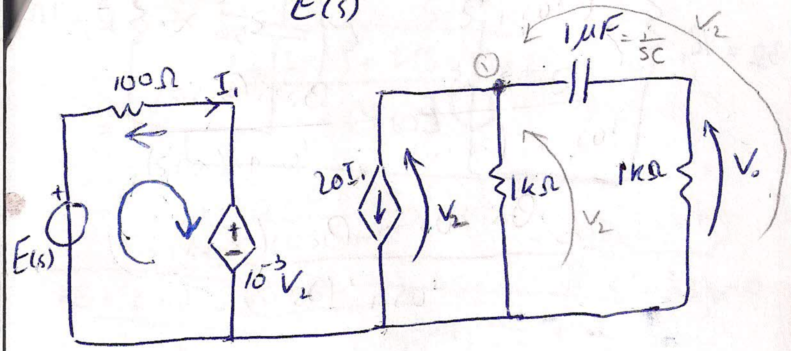
Evaluate $V_u(V_s)$, then repeat replacing L with C.



LAPLACE TRANSFORM II

Application to circuits

1 $H(s) = \dfrac{V_o(s)}{E(s)}$, $h(t) = ?$



KCL @ ①

$$20I_1 + \frac{V_2}{10^3} + \frac{V_2}{10^3 + \frac{10^6}{s}} = 0$$

$$I_1 = \frac{E(s) - 10^{-3}V_2}{100}$$ put in above

$$\cancel{20}\left(\frac{E(s) - 10^{-3}V_2}{\cancel{100}\ 5}\right) + \frac{V_2}{10^3} + \frac{V_2}{10^3 + \frac{10^6}{s}} = 0$$

$$\frac{E(s)}{5} - \frac{10^{-3}}{5}V_2 + \frac{V_2}{10^3} + \frac{V_2}{10^3 + \frac{10^6}{s}} = 0$$

$$V_2\left(\frac{1}{10^3} + \frac{1}{10^3 + \frac{10^6}{s}} - \frac{10^{-3}}{5}\right) = -\frac{E(s)}{5}$$

$$V_2\left(\frac{1}{10^3} + \frac{s}{s10^3 + 10^6} - \frac{10^{-3}}{5}\right) = \frac{-E(s)}{5}$$

$$V_2\left(\frac{5(s10^3+10^6) + 5s10^3 - (s10^3+10^6)}{\cancel{5}\,10^3(s10^3+10^6)}\right) = \frac{-E(s)}{\cancel{5}}$$

$$V_2\left(\frac{4(s10^3+10^6) + 5s10^3}{10^3(s10^3+10^6)}\right) = -E(s)$$

$$\boxed{V_2 = \frac{-E(s)\,10^3(s10^3+10^6)}{4(s10^3+10^6) + 5s10^3}}$$

$$V_o = \frac{10^3}{10^3 + \frac{10^6}{s}} V_2$$

$$V_o = \frac{s10^3}{\cancel{s10^3+10^6}}\left(\frac{-E(s)\,10^3\cancel{(s10^3+10^6)}}{4(s10^3+10^6) + 5s10^3}\right)$$

$$V_o = \frac{-E(s)\,s10^6}{4(s10^3+10^6) + 5s10^3}$$

$$H(s) = \frac{-s10^6}{4(s10^3+10^6) + 5s(10^3)}$$

$$= \frac{-s10^6}{4s10^3 + 4\,10^6 + 5s10^6}$$

$$= \frac{-s\,10^6}{9s10^3 + 4\,10^6} = \frac{-s10^3}{9s + 4\times10^3}$$

$$= \frac{-s10^3}{9\left(s + \frac{4\cdot10^3}{9}\right)}$$

$$H(s) = -\frac{10^3}{9}\left(\frac{s}{s + \frac{4}{9}\times10^3}\right)$$

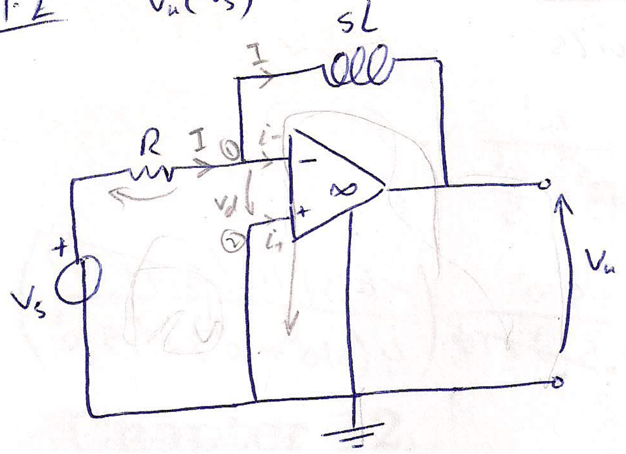
$$\begin{array}{r|l} & 1 \\ s + \frac{4}{9}\times10^3 & s \\ & s + \frac{4}{9}\times10^3 \\ \hline & -\frac{4}{9}\times10^3 \end{array}$$

$$= -\frac{10^3}{9}\left(1 + \frac{-\frac{4}{9}\times10^3}{s + \frac{4}{9}\times10^3}\right)$$

$$\boxed{H(s) = -\frac{10^3}{9} + \frac{+\frac{4}{81}10^6}{s + \frac{4}{9}\times10^3}}$$

$$\boxed{h(t) = -\frac{10^3}{9}\delta(t) + \frac{4}{81}\times10^6 e^{-\frac{4}{9}\times10^3 t}u(t)}$$

11.2 $V_u(V_s)$



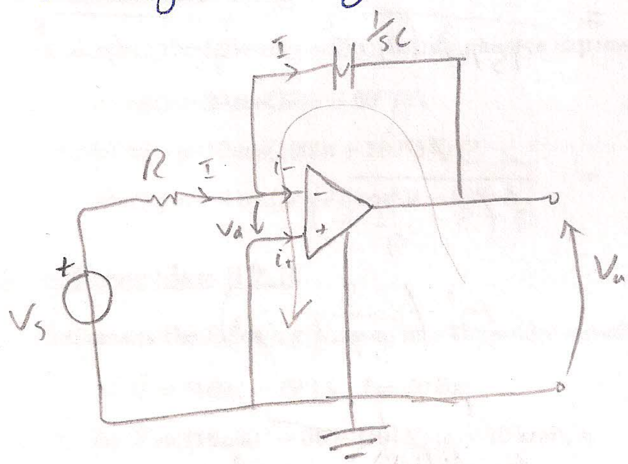
$I = \frac{V_s}{R}$ $\qquad V_s = IR$

$V_u + IsL + \cancel{V_d}^{\,0} = 0$

$V_u = -IsL$

$\boxed{V_u = -\frac{sL}{R} V_s}$

Replacing L by C



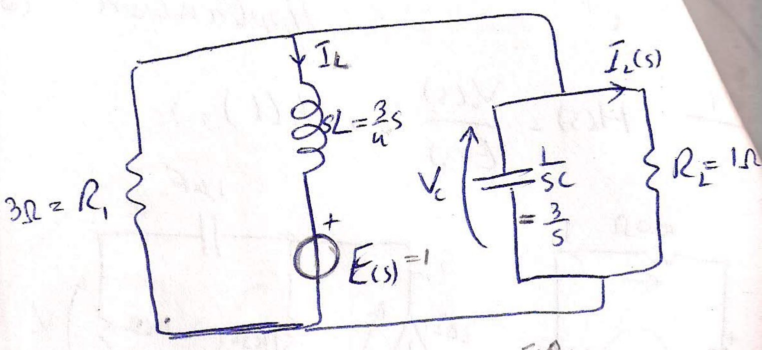
$I = \frac{V_s}{R}$

KVL $\quad V_u + \frac{I}{sC} + \cancel{V_d}^{\,0} = 0$

$V_u = -\frac{I}{sC}$

$\boxed{V_u = -\frac{1}{sCR} V_s}$

11.3 impulse response of i_L



$$V_c = \frac{\frac{E(s)}{\frac{3s}{4}}}{\frac{1}{3} + \frac{4}{3s} + \frac{s}{3} + 1} = \frac{\frac{E(s)\,4}{3s}}{\frac{s + 4 + s^2 + 3s}{3s}}$$

$$V_c = \frac{4E(s)}{s^2 + 4s + 4}$$

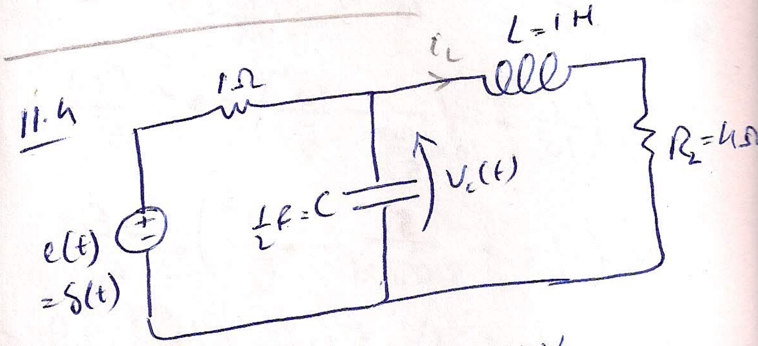
$$I_L(s) = \frac{4E(s)}{(s+2)^2}$$ ✓

$\boxed{H(s) = \frac{I_2(s)}{E(s)} ; E(s) = 1}$

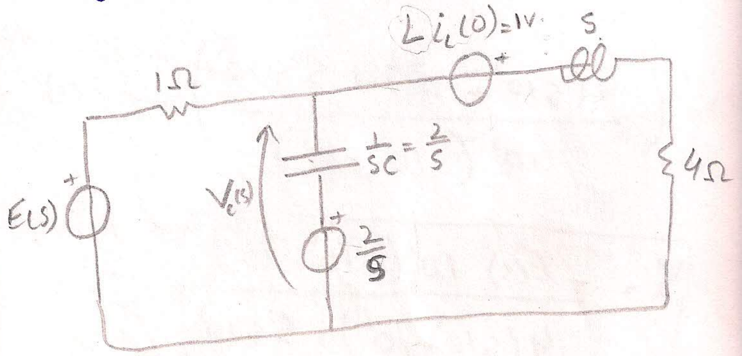
$$H(s) = \frac{4}{(s+2)^2}$$

$\boxed{h(t) = 4te^{-2t}u(t)}$

11.4



$i_L(0) = 1A, \quad V_c(0) = 2V$



$$V_c(s)=\frac{\frac{2/s}{2/s}+\frac{E(s)}{1}-\frac{1}{s+4}}{1+\frac{s}{2}+\frac{1}{s+4}}$$

$$=\frac{1+1-\frac{1}{s+4}}{\frac{2(s+4)+s(s+4)+2}{2(s+4)}}$$

$$=\frac{\frac{s+4+s+4-1}{s+4}}{\frac{2s+8+s^2+4s+2}{2\cancel{(s+4)}}}=\frac{2(2s+7)}{s^2+6s+10}$$

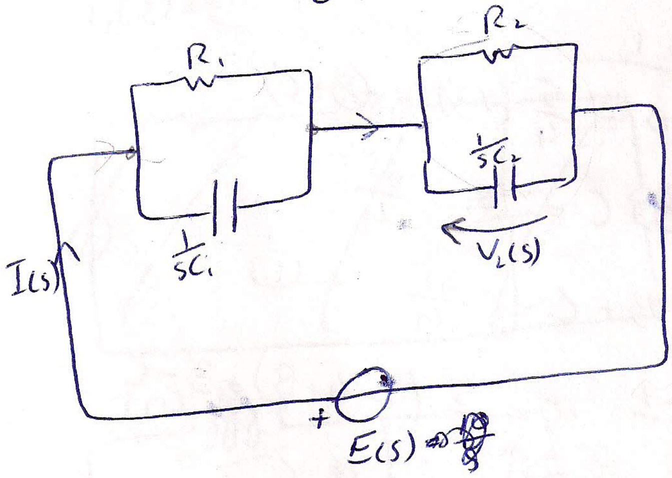
$$V_c(s)=\frac{4s+14}{s^2+2(3)s+(3)^2+1}=\frac{4s+12+2}{(s+3)^2+(1)^2}$$

$$V_c(s)=\frac{4(s+3)}{(s+3)^2+(1)^2}+\frac{2}{(s+3)^2+(1)^2}$$

$$\boxed{v_c(t)=\left[4e^{-3t}\cos(t)+2e^{-3t}\sin(t)\right]u(t)}$$

11.5 $H(s)=\frac{I(s)}{E(s)}$

$H_2(s)=\frac{V_2(s)}{E(s)}$



$$\frac{R_1\frac{1}{sC_1}}{R_1+\frac{1}{sC_1}}=\frac{R_1/sC_1}{\frac{sR_1C_1+1}{sC_1}}=\frac{R_1}{1+sR_1C_1}$$

$$\frac{R_2\frac{1}{sC_2}}{R_2+\frac{1}{sC_2}}=\frac{R_2/sC_2}{\frac{sR_2C_2+1}{sC_2}}=\frac{R_2}{1+sR_2C_2}$$

$$Z_{eq}=\frac{R_1}{1+sR_1C_1}+\frac{R_2}{1+sR_2C_2}$$

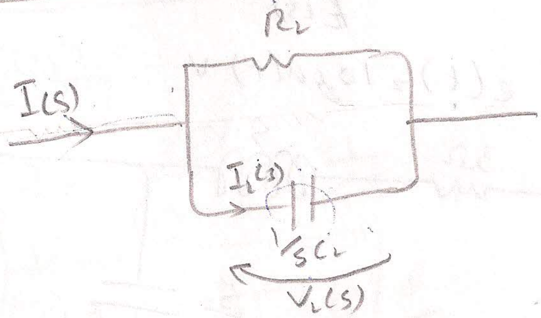
$$=\frac{R_1(1+sR_2C_2)+R_2(1+sR_1C_1)}{(1+sR_1C_1)(1+sR_2C_2)}$$

$$I(s)=\frac{E(s)}{Z_{eq}}$$

$$I(s)=\frac{E(s)(1+sR_1C_1)(1+sR_2C_2)}{R_1+sR_1R_2C_2+R_2+sR_1R_2C_1}$$

$$H(s)=\frac{I(s)}{E(s)}$$

$$\boxed{H(s)=\frac{(1+sR_1C_1)(1+sR_2C_2)}{R_1+R_2+sR_1R_2(C_1+C_2)}}$$



$$I_2(s)=\frac{R_2}{R_2+\frac{1}{sC_2}}I(s)$$

$$=\frac{sR_2C_2}{1\cancel{+sC_2R_2}}\left(\frac{E(s)(1+sR_1C_1)\cancel{(1+sR_2C_2)}}{R_1+R_2+sR_1R_2(C_1+C_2)}\right)$$

$$I_2(s)=\frac{E(s)\,sR_2C_2(1+sR_1C_1)}{R_1+R_2+sR_1R_2(C_1+C_2)}$$

$$V_2(s)=\frac{E(s)\cancel{s}R_2\cancel{C_2}(1+sR_1C_1)}{R_1+R_2+sR_1R_2(C_1+C_2)}\cdot\frac{1}{\cancel{sC_2}}$$

$$\boxed{V_2(s)=\frac{E(s)(1+sR_1C_1)R_2}{R_1+R_2+sR_1R_2(C_1+C_2)}}$$

$$\boxed{H_2(s)=\frac{(1+sR_1C_1)R_2}{R_1+R_2+sR_1R_2(C_1+C_2)}}$$

For relationship

$$(1+sC_1R_1)R_2 = 0$$

$$1+sC_1R_1 = 0$$

$$s = -\frac{1}{C_1R_1}$$

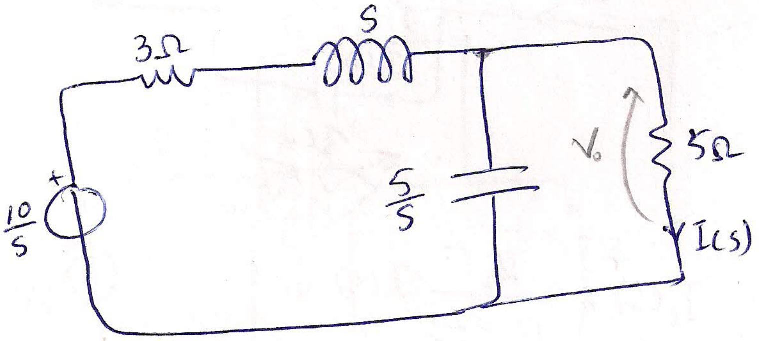
$$1+s\frac{R_1R_2}{R_1+R_2}(C_1+C_2) = 0$$

$$+\frac{R_1R_2}{C_1R_1(R_1+R_2)}(C_1+C_2) = 1$$

$$\boxed{\frac{R_2}{R_1+R_2} = \frac{C_1}{C_1+C_2}}$$

11.6 $H(s) = \frac{I(s)}{E(s)}$, $i(t)$

$e(t) = 10u(t)$ V $\quad E(s) = \frac{10}{s}$



$$V_0(s) = \frac{\frac{10/s}{3+s}}{\frac{1}{s+3}+\frac{s}{5}+\frac{1}{5}}$$

$$= \frac{\frac{10}{\cancel{s(3+s)}}}{\frac{5+s(3+s)+s+3}{5\cancel{(s+3)}}}$$

$$= \frac{10.5}{s(5+3s+s^2+s+3)}$$

$$= \frac{50}{s(s^2+4s+8)}$$

$$I(s) = \frac{V_0}{5} = \frac{10}{s(s^2+4s+8)}$$

$$H(s) = \frac{\frac{\cancel{10}/s}{s^2+4s+8}}{\cancel{10}/s}$$

$$H(s) = \frac{1}{s^2+2(s)2+(2)^2+4}$$

$$\boxed{H(s) = \frac{1}{(s+2)^2+4}}$$

or

$$\boxed{H(s) = \frac{1}{s^2+4s+8}}$$

$$I(s) = \frac{10}{s(s^2+4s+8)}$$

$$\frac{10}{s(s^2+4s+8)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+8}$$

$$10 = A(s^2+4s+8) + (Bs+C)s$$

$s=0$

$$10 = 8A \Rightarrow \boxed{A = \frac{5}{4}}$$

$s=1$

$$10 = \frac{5}{4}(13) + (B+C)$$

$$B+C = 10 - \frac{65}{4}$$

$$B+C = -\frac{25}{4} \quad —①$$

$s=-1$ $\quad 10 = \frac{5}{4}(1-4+8)+B-C$

$$10 = \frac{25}{4}+B-C$$

$$B-C = \frac{15}{4} \quad —②$$

①+②

$$2B = \frac{-10}{4} \qquad C = -\frac{25}{4}+\frac{5}{4}$$

$$\boxed{B = -\frac{5}{4}} \qquad \boxed{C = -5}$$

$$(s) = \frac{5/4}{s} + \frac{-\frac{5}{4}s - 5}{(s+2)^2+4}$$

$$= \frac{5}{4}\frac{1}{s} + \frac{\frac{-5s-20}{4}}{(s+2)^2+4}$$

$$= \frac{5}{4}\frac{1}{s} - \frac{5}{4}\left(\frac{s+4}{(s+2)^2+4}\right)$$

$$= \frac{5}{4}\frac{1}{s} - \frac{5}{4}\left(\frac{(s+2)+2}{(s+2)^2+4}\right)$$

$$= \frac{5}{4}\frac{1}{s} - \frac{5}{4}\left[\frac{s+2}{(s+2)^2+4} + \frac{2}{(s+2)^2+4}\right]$$

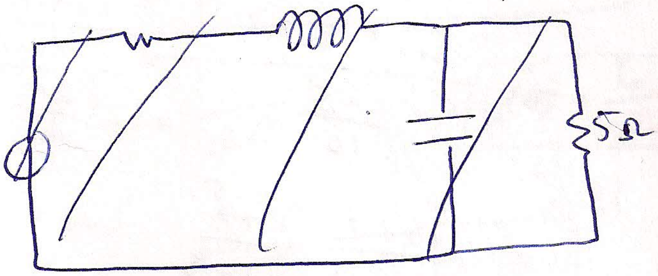
$$i(t) = \frac{5}{4}\left[u(t) - \left(e^{-2t}\cos(2t) + e^{-2t}\sin(2t)\right)u(t)\right]$$

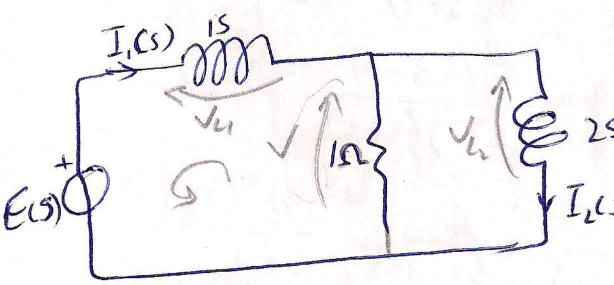
$$\boxed{i(t) = \frac{5}{4}\left[1 - e^{-2t}\left(\cos(2t) + \sin(2t)\right)\right]u(t)}$$

11.7 $\quad H_1(s) = \frac{I_1(s)}{E(s)}, \quad H_2(s) = \frac{I_2(s)}{E(s)}$

$e(t) = 0V$

$i_1(0) = 2A, \quad i_2(0) = 1A$





$$V = \frac{E(s)/s}{\frac{1}{s} + 1 + \frac{1}{2s}}$$

$$V = \frac{E(s)/s}{\frac{2+2s+1}{2s}} = \frac{2E(s)}{2s+3}$$

$$V + V_{L1} - E(s) = 0$$

$$V_{L1} = E(s) - \frac{2E(s)}{2s+3}$$

$$V_{L1} = \frac{E(s)(2s+3) - 2E(s)}{2s+3}$$

$$V_{L1} = \frac{E(s)(2s+3-2)}{2s+3}$$

$$I_1(s) = \frac{E(s)(2s+1)}{s(2s+3)} \quad ?$$

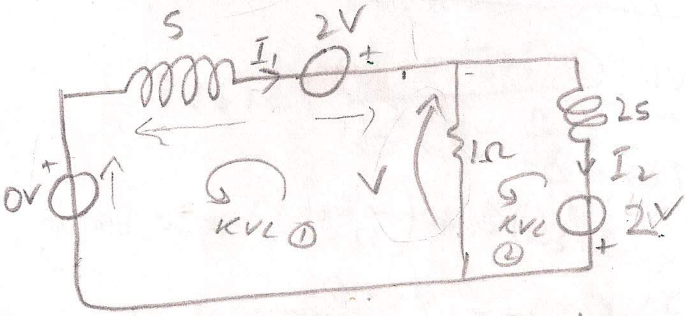
$$H_1(s) = \frac{\cancel{2}(s+\frac{1}{2})}{2s(s+3/2)}$$

$$\boxed{H_1(s) = \frac{s+1/2}{s(s+3/2)}}$$

$$V_{L2} = V$$

$$I_2(s) = \frac{\cancel{2}E(s)}{(2s+3)} \cdot \frac{1}{\cancel{2}s}$$

$$\boxed{H_2(s) = \frac{1}{2s(s+3/2)}}$$



$$V = \frac{2/s - \frac{1}{s}}{\frac{1}{s} + 1 + \frac{1}{2s}} = \frac{\frac{2-1}{s}}{\frac{2+2s+1}{2s}}$$

$$\boxed{V = \frac{2}{2s+3}V}$$

KVL ①

$I_1 s - 2 + V = 0$

$I_1(s)\, s - 2 + \frac{2}{2s+3} = 0$

$I_1(s) = \frac{4s+6-2}{2s+3}$

$I_1(s) = \frac{4(s+1)}{2s(s+\frac{3}{2})}$

$I_1(s) = \frac{2(s+1)}{s(s+\frac{3}{2})}$

$\frac{2(s+1)}{s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+3/2}$

$2(s+1) = A(s+\frac{3}{2}) + B(s)$

$s = 0$

$2 = \frac{3}{2} A \Rightarrow \boxed{A = \frac{4}{3}}$

$s = -\frac{3}{2}$

$2(-\frac{3}{2}+1) = B(-\frac{3}{2})$

$-1 = B(-\frac{3}{2}) \Rightarrow \boxed{B = \frac{+2}{3}}$

$I_1(s) = \frac{4/3}{s} + \frac{2/3}{s+3/2}$

$\boxed{i_1(t) = \left(\frac{4}{3} + \frac{2}{3} e^{-\frac{3}{2}t}\right)u(t)\ A}$

KVL ②

$I_2(2s) - 2 - V = 0$

$I_2(s)(2s) = 2 + \frac{2}{2s+3}$

$I_2(s) = \frac{4s+6+2}{2s(2s+3)}$

$I_2(s) = \frac{2(s+2)}{s(s+3/2)}$

$\frac{(s+2)}{s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+\frac{3}{2}}$

$(s+2) = A(s+\frac{3}{2}) + B(s)$

$s = 0$

$2 = \frac{3}{2} A \Rightarrow \boxed{A = \frac{4}{3}}$

$s = -\frac{3}{2}$

$-\frac{3}{2} + 2 = -B\frac{3}{2}$

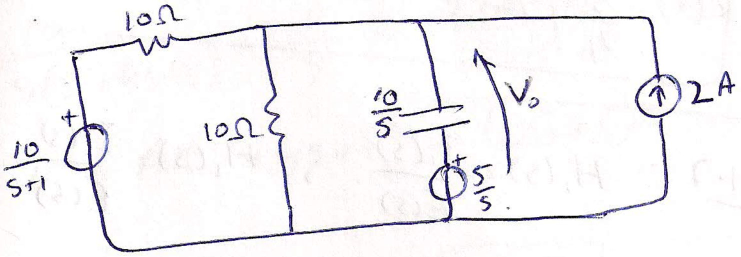
$\frac{1}{2} = -B\frac{3}{2} \Rightarrow \boxed{B = \frac{-1}{3}}$

$I_2(s) = \frac{4/3}{s} + \frac{-1/3}{s+3/2}$

$\boxed{i_2(t) = \left(\frac{4}{3} - \frac{1}{3} e^{-3/2\, t}\right)u(t)\ A}$

11.8 $V_o(0) = 5V$

$V_o(t) = ?$



$V_o(s) = \frac{\frac{10/s+1}{10} + \frac{5/s}{10/s} + 2}{\frac{1}{10} + \frac{1}{10} + \frac{s}{10}}$

$= \frac{\frac{1}{s+1} + \frac{1}{2} + 2}{\frac{2+s}{10}} = \frac{\frac{2+s+1+4s+4}{2(s+1)}}{\frac{2+s}{10}}$

$= \frac{5s+7}{2(s+1)} \times \frac{\cancel{10}^{\,5}}{2+s}$

$V_o(s) = \frac{5(5s+7)}{(s+1)(s+2)}$

$\frac{5(5s+7)}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$5(5s+7) = A(s+2) + B(s+1)$

-1

$(2) = A \Rightarrow A = 10$

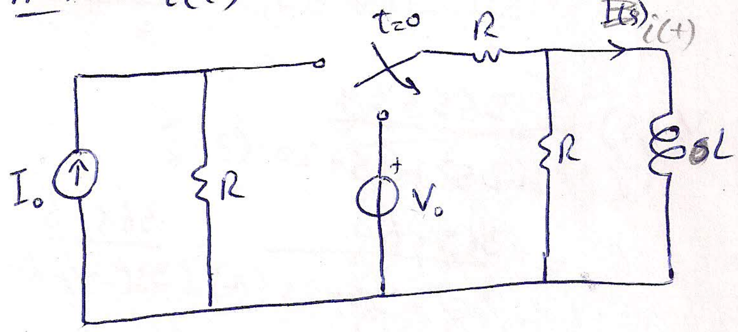
$s = -2$

$5(-3) = B(-1) \Rightarrow B = 15$

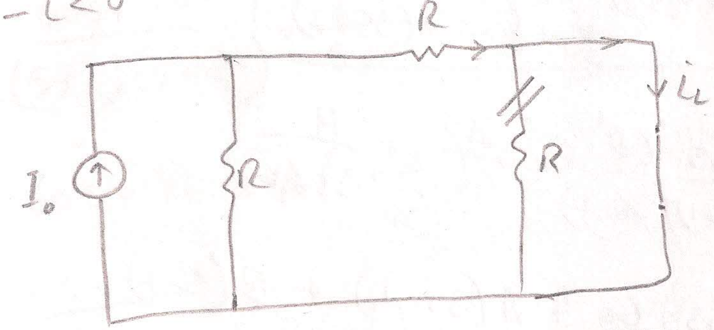
$$V_o(s) = \frac{10}{s+1} + \frac{15}{s+2}$$

$$\boxed{v_o(t) = \left(10e^{-t} + 15e^{-2t}\right)u(t)\ V}$$

11.9 $i(t)$ $\forall t$

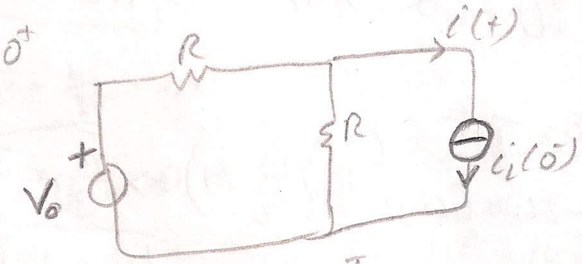


$- t<0$



$$i_L(0^-) = \frac{R}{2R} I_o \Rightarrow \boxed{i_L(0^-) = \frac{I_o}{2}\ A}$$

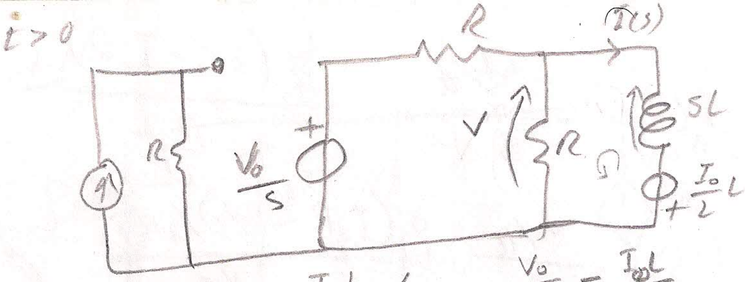
$- t \to 0^+$



$$i_L(0^+) = i_L(0^-) = \frac{I_o}{2}$$

(Steady-state)

$t>0$



$$V = \frac{\frac{V_o/s}{R} - \frac{I_oL}{2}/sL}{\frac{1}{R} + \frac{1}{R} + \frac{1}{sL}} = \frac{\frac{V_o}{sR} - \frac{I_oL}{2sL}}{\frac{2sL+R}{RsL}}$$

$$V = \frac{\frac{2V_oL - I_oLR}{2sRL}}{\frac{2sL+R}{RsL}} = \frac{L2V_o - I_oLR}{2(2sL+R)}$$

$$V = \frac{L(2V_o - I_oR)}{2(R+2sL)}$$

KVL

$$I(s)\,sL = \frac{I_oL}{2} + \frac{L}{2}\left(\frac{2V_o - I_oR}{R+2sL}\right)$$

$$I(s) = \frac{\cancel{L}}{2s\cancel{L}}\left(I_o + \frac{2V_o - I_oR}{R+2sL}\right)$$

$$I(s) = \frac{\cancel{I_oR} + I_o2sL + 2V_o - \cancel{I_oR}}{2s(R+2sL)}$$

$$I(s) = \frac{I_o2sL + 2V_o}{2s(R+2sL)}$$

$$I(s) = \frac{2(V_o + I_osL)}{2Ls\left(s + \frac{R}{2L}\right)}$$

$$V_o + I_osL = A\left(s + \frac{R}{2L}\right) + B(s)$$

$- s = 0$

$$V_o = A\frac{R}{2L} \Rightarrow A = \frac{2V_oL}{R}\cdot\frac{1}{2L}$$

$$\boxed{A = \frac{V_o}{R}}$$

$- s = -\frac{R}{2L}$

$$-\frac{R}{2L}B = V_o - \frac{I_oL\cdot R}{2L}$$

$$B = -\frac{2V_o - I_oR}{\cancel{2}}\cdot\frac{\cancel{2}L}{R}$$

$$B = -\frac{L}{R}\left(\frac{2V_o - I_oR}{2L}\right)$$

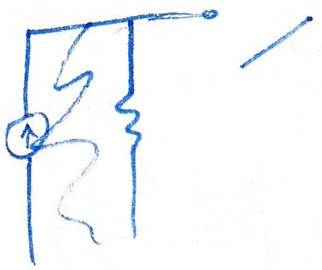
$$B = \frac{I_oR - 2V_o}{2R}$$

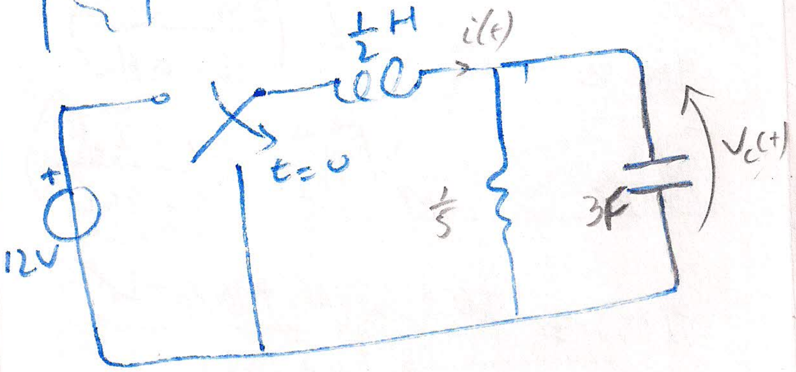
$$I(s) = \frac{2V_0 L/R}{s} + \frac{-\frac{L}{R}(2V_0 - I_0 R)}{s+\frac{R}{2L}}$$

$$I(s) = \frac{V_0/R}{s} + \frac{(I_0R-2V_0)/2R}{s+R/2L} \quad ?$$

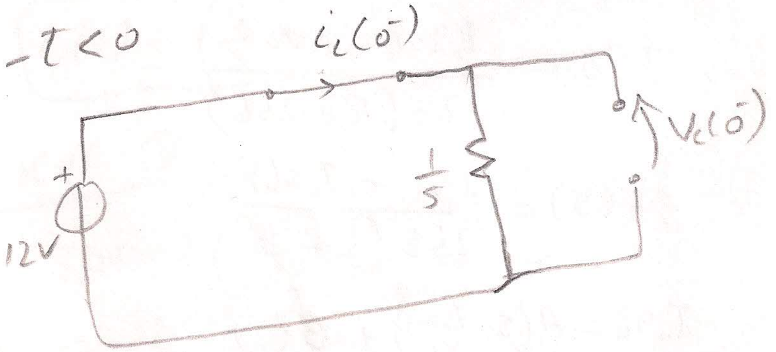
$$\boxed{i(t) = \frac{V_0}{R}u(t) + \left(\frac{I_0}{2}-\frac{V_0}{R}\right)e^{-\frac{R}{2L}t}u(t)}$$

11.10





- t<0



$V_c(0^-) = 12V$

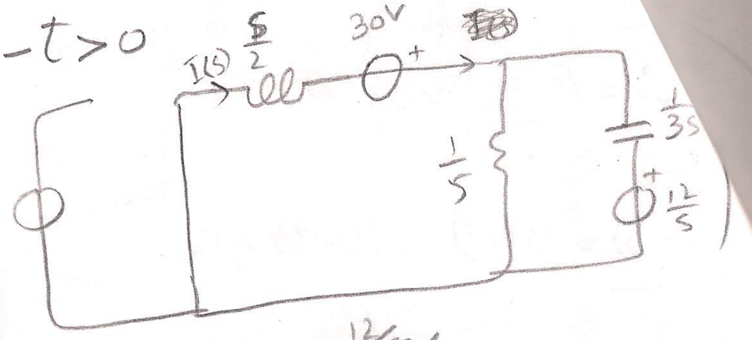
$i_L(0^-) = \frac{12}{1/5} = 60A$

- t→0⁺

$V_c(0^+) = V_c(0^-) = 12V$

$i_L(0^+) = i_L(0^-) = 60A$

- t>0



$$V_c(s) = \frac{\frac{30}{s/2} + \frac{12/s}{1/3s}}{\frac{1}{s/2} + \frac{1}{1/5} + \frac{1}{1/3s}}$$

$$V_c(s) = \frac{\frac{60}{s}+36}{\frac{2}{s}+5+3s} = \frac{\frac{60+36s}{s}}{\frac{2+5s+3s^2}{s}}$$

$$V_c(s) = \frac{36s+60}{3s^2+5s+2} = \frac{36s+60}{3s^2+3s+2s+2} = \frac{36s+60}{3s(s+1)+2(s+1)}$$

$$\boxed{V_c(s) = \frac{36s+60}{(s+1)(3s+2)}}$$

$$\frac{36s+60}{(s+1)(3s+2)} = \frac{A}{s+1} + \frac{B}{3s+2}$$

$$36s+60 = A(3s+2) + B(s+1)$$

- $s=-1$

$-36+60 = -A \Rightarrow \boxed{A=-24}$

- $s = -\frac{2}{3}$

$-24+60 = B\left(-\frac{2}{3}+1\right) \Rightarrow \frac{B}{3} = 36$ $\boxed{B=108}$

$$V_c(s) = \frac{-24}{s+1} + \frac{\overset{36}{\cancel{108}}}{\cancel{3}\left(s+\frac{2}{3}\right)}$$

$$\boxed{V_c(t) = \left(-24e^{-t} + 36\, e^{\frac{2}{3}t}\right)u(t)\ V}$$

$$I(s)\,\frac{s}{2} + 30 - \frac{36s+60}{(s+1)(3s+2)} = 0$$

$$I(s)\,\frac{s}{2} = 30 - \frac{36s+60}{(s+1)(3s+2)}$$

$$I(s) = \frac{60}{s} - \frac{72s+120}{s(s+1)(3s+2)}$$

$$I(s) = \frac{180s^2+300s+\cancel{120}-72s-\cancel{120}}{s(s+1)(3s+2)}$$

$$I(s) = \frac{180s^2+228s}{s(s+1)(3s+2)}$$

$$I(s) = \frac{\cancel{3s}(60s+76)}{\cancel{3s}(s+1)(s+\frac{2}{3})}$$

$$I(s) = \frac{60s+76}{(s+1)(s+\frac{2}{3})}$$

$$\frac{60s+76}{(s+1)(s+\frac{2}{3})} = \frac{A}{s+1} + \frac{B}{s+\frac{2}{3}}$$

$$60s+76 = A\left(s+\frac{2}{3}\right) + B(s+1)$$

$s=-1$

$$-60+76 = A\left(-\frac{1}{3}\right) \Rightarrow -\frac{A}{3} = 16$$

$$\boxed{A=-48}$$

$s=-\frac{2}{3}$

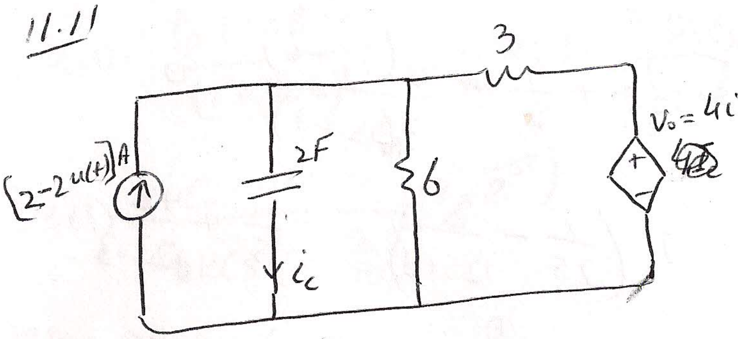
$$-40+76 = 0 + B\left(\frac{1}{3}\right) \Rightarrow \frac{B}{3} = 36$$

$$\boxed{B=108}$$

$$I(s) = \frac{-48}{s+1} + \frac{108}{s+\frac{2}{3}}$$

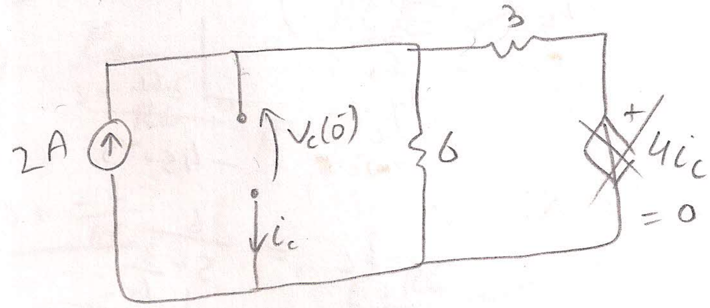
$$\boxed{i(t) = \left(-48e^{-t} + 108e^{-\frac{2}{3}t}\right)u(t)\ A}$$

11.11

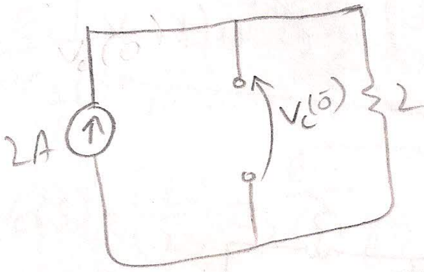


For initial conditions

$t<0 \qquad u(t)=0$

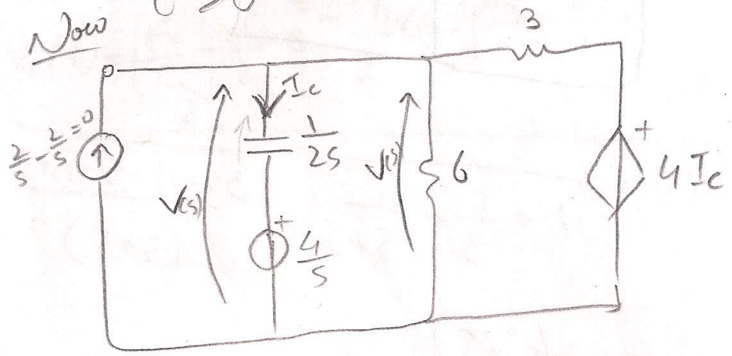


$i_c = 0$



$$V_c(0^-) = 2\times 2 = \boxed{4\,V}$$

Now $t>0$



$$V = \frac{\frac{4/s}{1/2s} + \frac{4I_c}{3}}{2s+\frac{1}{6}+\frac{1}{3}} = \frac{\frac{3\times 8 + 4I_c}{3}}{\frac{12s+1+2}{\cancel{6}\,2}}$$

$$\boxed{V = \frac{8(I_c+6)}{3(4s+1)}}$$

KVL

$$\frac{I_c}{2s} + \frac{4}{s} - \frac{8(I_c+6)}{3(4s+1)} = 0$$

$$I_c\left(\frac{1}{2s} - \frac{8}{12s+3}\right) = -\frac{4}{s} + \frac{48}{12s+3}$$

$$I_c\left(\frac{12s+3-16s}{2s(\cancel{12s+3})}\right) = \frac{-48s-12+48s}{s(\cancel{12s+3})}$$

$$I_c\left(\frac{-4s+3}{2}\right) = -12$$

$$I_c = \frac{-24}{-4s+3}$$

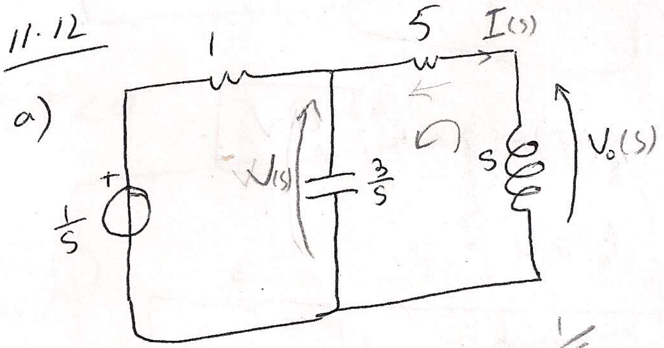
$$I_c = \frac{6}{s-\frac{3}{4}}$$

$$\boxed{i_c(t) = 6e^{\frac{3}{4}t}u(t)\ A}$$

$$\boxed{v_o(t) = 24e^{3/4\,t}u(t)\ V}$$

11.12

a)



$$V(s) = \frac{\frac{1}{s}}{1+\frac{s}{3}+\frac{1}{s+5}} = \frac{1/s}{\frac{3s+15+s^2+5s+3}{3(s+5)}}$$

$$V(s) = \frac{3(s+5)}{s(s^2+8s+18)}$$

$$I(s) = \frac{3\cancel{(s+5)}}{s(s^2+8s+18)}\cdot\frac{1}{\cancel{s+5}}$$

$$I(s) = \frac{3}{s(s^2+8s+18)}\ A$$

$$V_o(s) + \frac{3\cdot 5}{s(s^2+8s+18)} - \frac{3(s+5)}{s(s^2+8s+18)}$$

$$V_o(s) = \frac{3(s+5)-15}{s(s^2+8s+18)}$$

$$V_o(s) = \frac{3s}{s(s^2+8s+18)}$$

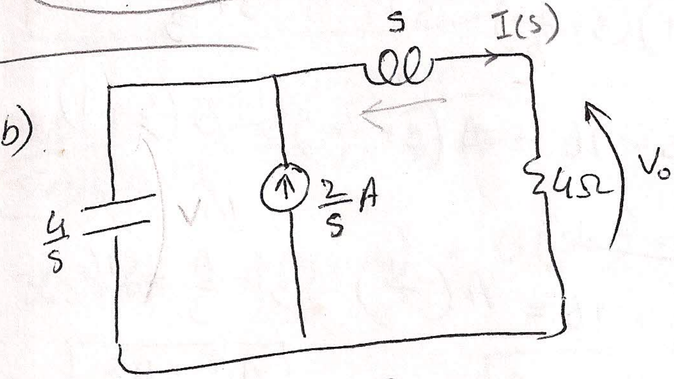
$$V_o(s) = \frac{3}{s^2+8s+18}$$

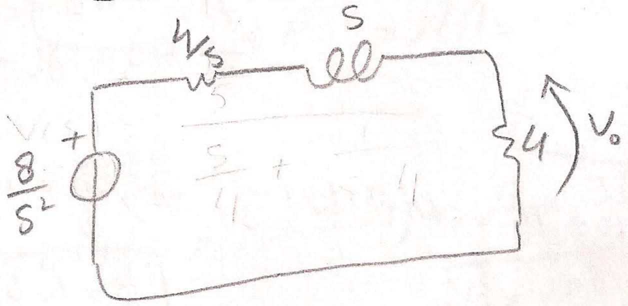
$$= \frac{3}{(s+4)^2+(\sqrt{2})^2}$$

$$= \frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{(s+4)^2+(\sqrt{2})^2}$$

$$\boxed{v_o(t) = \frac{3}{\sqrt{2}}e^{-4t}\sin(\sqrt{2}t)\,u(t)\ V}$$

b)





$$V_o(s) = \frac{4}{4+s+\frac{4}{s}}\cdot\frac{8}{s^2}$$

$$V_o(s) = \frac{4}{\frac{4s+s^2+4}{s}}\cdot\frac{8}{s^2} = \frac{32}{s(s+2)^2}$$

$$V_o(s) = \frac{32}{s(s+2)^2}$$

$$\frac{32}{s(s+2)^2} = \frac{A}{s} + \frac{B}{(s+2)^2} + \frac{C}{s+2}$$

$$32 = A(s+2)^2 + B(s) + C(s+2)s$$

– $s=0$

$$32 = 4A \Rightarrow \boxed{A=8}$$

– $s=-2$

$$32 = -2B \Rightarrow \boxed{B=-16}$$

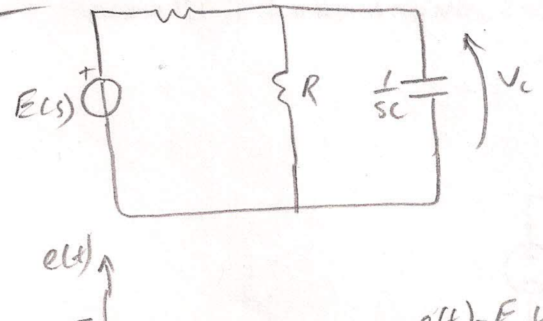
$$C = \lim_{s\to -2} \frac{d}{ds}\left(\cancel{(s+2)^2}\,\frac{32}{s\cancel{(s+2)^2}}\right)$$

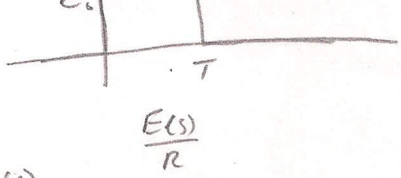
$$C = \lim_{s\to -2} -32 s^{-2} \Rightarrow \boxed{C=-8}$$

$$V_o(s) = \frac{8}{s} - \frac{16}{(s+2)^2} + \frac{-8}{s+2}$$

$$\boxed{v_o(t) = \left(8 - 16te^{-2t} - 8e^{-2t}\right)u(t)\ V}$$

11.13





$$e(t) = E_0 u(t) - E_0 u(t-T)$$

$$E(s) = \frac{E_0}{s} - \frac{E_0 e^{-sT}}{s}$$

$$V_c(s) = \frac{\frac{E(s)}{R}}{\frac{1}{R}+\frac{1}{R}+sC} = \frac{\frac{E(s)}{R}}{\frac{2+RsC}{R}}$$

$$V_c(s) = \frac{E(s)}{2+RsC}$$

$$V_c(s) = \frac{\frac{E_0}{s}(1-e^{-sT})}{(2+RsC)} \Rightarrow V_c(s) = \frac{E_0(1-e^{-sT})}{s(2+RsC)}$$

$$V_c(s) = \frac{E_0}{RCs\left(s+\frac{2}{RC}\right)}(1-e^{-sT})$$

$$\frac{E_0}{RCs\left(s+\frac{2}{RC}\right)} = \frac{A}{s} + \frac{B}{s+\frac{2}{RC}}$$

– $s=0$

$$A = \left.\frac{E_0}{s+\frac{2}{RC}}\right|_0 = \frac{RCE_0}{2}\ ✓$$

– $s=\frac{-2}{RC}$

$$B = \left.\frac{E_0}{s}\right|_{-\frac{2}{RC}} = \frac{E_0 RC}{-2}\ ✓ \quad \frac{E_0}{2}$$

$$\frac{E_0}{RCs\left(s+\frac{2}{RC}\right)} = \frac{\cancel{RC}}{\cancel{RC}}\left(\frac{\frac{E_0}{2}}{s} - \frac{E_0/2}{s+\frac{2}{RC}}\right)$$

$$V_c(s) = \left[\frac{E_0}{2s} - \frac{E_0}{2\left(s+\frac{2}{RC}\right)}\right](1-e^{-sT})$$

$$v_c(t) = \left(\frac{E_0}{2}u(t) - \frac{E_0}{2}e^{-\frac{2}{RC}t}u(t)\right)(1-e^{-sT})$$

$$\boxed{v_c(t) = \left[\frac{E_0}{2} - \frac{E_0}{2}e^{-\frac{2}{RC}t}\right]u(t) - \left[\frac{E_0}{2} - \frac{E_0}{2}e^{-\frac{2}{RC}(t-T)}\right]u(t-T)}$$

Rectangular Form: $\hat{x} = a \mp jb$

Magnitude $= \sqrt{a^2+b^2} = |\hat{x}|$

Phase Angle $= \angle\hat{x} = \tan^{-1}\frac{b}{a}$

if $\hat{x} = a+jb$, $\tan^{-1}\frac{b}{a} = \theta^\circ$

$\hat{x} = a-jb$, $\tan^{-1}\frac{-b}{a} = \theta^\circ$

$\hat{x} = -a+jb$, $\left(\tan^{-1}\frac{b}{-a}\right)+180^\circ = \theta^\circ$

$\hat{x} = -a-jb$, $\left(\tan^{-1}\frac{b}{a}\right)-180^\circ = \theta^\circ$

Phasor Form $\hat{x} = |\hat{x}|\angle\theta^\circ$

$\hat{x} = |\hat{x}|e^{j\theta}$ (Polar Form)

Sinusoidal Form $= |\hat{x}|\cos(wt \mp \theta^\circ) = \hat{x}$ (i)

w = Angular Frequency $= 2\pi f_0$

$\angle = \theta^\circ$

$\hat{x} = |\hat{x}| \sin(wt \mp \theta^\circ)$ (ii)

$\angle = \theta^\circ - 90^\circ$

12.1

a) $(2+j3)(-4-j5) = 7 - j22$

$= -8 - 10j - 12j - j^2 15$

$= -8 + 15 - 22j = \boxed{7 - j22}$

$2+j3 \qquad -4-j5$

$r_1 = \sqrt{13} \qquad r_2 = \sqrt{41}$

$\theta_1 = 56.309^\circ \qquad \theta_2 = 51 + 180$

$\theta_2 = 231.34$

$(2+j3)(-4-j5) = \sqrt{13 \times 41} \angle 56.309 + 231.34$

$= 23.086\, e^{j287.649}$

$= 23.086\left(\cos(287.64) + j\sin(287.64)\right)$

$= 7 - 22j$

b) $\dfrac{-4-j5}{2+j3} = -\dfrac{23}{13} + j\dfrac{2}{13}$

a) $(2+j3)(-4-j5) = 7 - j22$

1) $-8 - 10j - 12j - j^2 15$

$= -8 - 22j + 15$

$= 7 - 22j = R.H.S$

2) $2+j3 \qquad \underline{x} + yj$
$\qquad -4-j5$

$r_1 = \sqrt{4+9} \qquad r_2 = \sqrt{16+25}$

$r_1 = \sqrt{13} \qquad r_2 = \sqrt{41}$

$\theta_1 = \tan^{-1}\dfrac{3}{2} \qquad \theta_2 = \tan^{-1}\dfrac{-5}{-4} + 180$

$\theta_1 = 56.309 \qquad \theta_2 = 231.34$

$(2+j3)(-4-j5) = \sqrt{13} \cdot \sqrt{41}\, e^{j(56.309}$

$= 23.086\, e^{j(287.649)}$

$= 23.086\left(\cos(287.649) + j\sin(287.6\ldots\right)$

$= 6.99 - j21.99 = R.H.S$

b) $\dfrac{-4-j5}{2+j3} = -\dfrac{23}{13} + j\dfrac{2}{13}$

$(-4-j5) \qquad 2+j3$

$r_1 = \sqrt{16+25} \qquad r_2 = \sqrt{4+9}$

$r_1 = \sqrt{41} \qquad r_2 = \sqrt{13}$

$\theta_1 = 231.34 \qquad \theta_2 = 56.309$

$\dfrac{-4-j5}{2+j3} = \dfrac{\sqrt{41}\, e^{231.34}}{\sqrt{13}\, e^{56.309}}$

$= 1.77\, e^{175.031}$

$= 1.77\left(\cos(175.031) + j\sin(175.031)\right)$

$= -1.7633 + j0.1533 =$

$= -\dfrac{23}{13} + j\dfrac{2}{13}$

$$= \frac{5/4}{s} + \frac{-\frac{5}{4}s - \frac{25}{\cancel{8}}}{(s+2)^2+(2)^2}$$

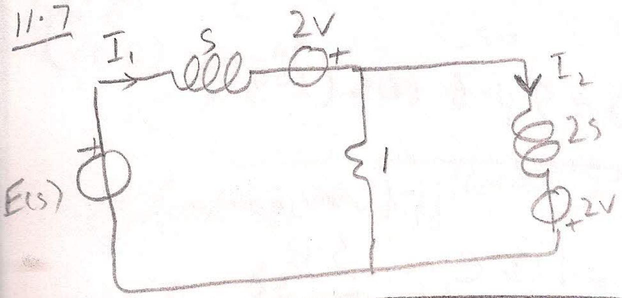
$$(s) = \frac{5/4}{s} + \frac{\frac{-5s-20}{4}}{(s+2)^2+(2)^2}$$

$$= \frac{5/4}{s} + -\frac{5}{4}\,\frac{s+4}{(s+2)^2+(2)^2}$$

$$= \frac{5}{4}\left[\frac{1}{s} - \left(\frac{s+2}{(s+2)^2+(2)^2} + \frac{2}{(s+2)^2+(2)^2}\right)\right]$$

$$= \frac{5}{4}\left(u(t) - \left(e^{-2t}\cos(2t) + e^{-2t}\sin(2t)\right)u(t)\right)$$

11.7



12.2

a) $v(t) = 20\cos(150t - 60^\circ)$ V

$$V(t) = 20\,e^{-60j}$$

$$= 20\left(\cos(-60) + j\sin(-60^\circ)\right)$$

$$= 10 - j\frac{\sqrt{3}}{2}\cdot 10$$

$$\boxed{\hat{V} = \frac{10}{\sqrt{2}} - j\sqrt{\frac{3}{2}}\,10}$$

b) $v(t) = 10\cos(1000t + 180^\circ)$ V

$$v(t) = 10\,e^{180j}$$

$$= 10\left(\cos(180^\circ) + j\sin(180^\circ)\right)$$

$$= -10$$

$$\boxed{\hat{V} = -\frac{10}{\sqrt{2}}\text{ V}}$$

c) $i(t) = -4\cos(\underbrace{3t}_{\omega t+\varphi}) + 3\cos(\underbrace{3t - 90^\circ}_{\omega t+\varphi})$

$$i(t) = -4e^{0j} + 3e^{-90j}$$

$$= -4\left(\cos(0^\circ) + j\sin 0^\circ\right) + 3\left(\cos(-90^\circ) + j\sin(-90^\circ)\right)$$

$$i(t) = -4 - 3j$$

$$\hat{I} = \frac{i(t)}{\sqrt{2}} = -\frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}j\text{ A}$$

12.3 a) $\hat{V} = (169\angle -45^\circ)$ V

$f = 60\text{Hz}$ $\omega = 2\pi f = 120\pi$

$$\frac{v(t)}{\sqrt{2}} = 169\,e^{-45j}$$

$$= 169\left(\cos(-45) + j\sin(-45)\right)$$

$$\frac{v(t)}{\sqrt{2}} = \frac{169}{\sqrt{2}} - j\frac{169}{\sqrt{2}}$$

$$v(t) = 169 - j169$$

$$\frac{v(t)}{\sqrt{2}} = 169\cos(120\pi t - 45^\circ)$$

$$\boxed{v(t) = 169\sqrt{2}\cos(120\pi t - 45^\circ)\text{ V}}$$

$\omega = 10k$ rad/s

b) $\hat{V} = (10\angle 90^\circ + 66 - j10)$ V

$$\frac{v(t)}{\sqrt{2}} = 10e^{90j} + 66 - j10$$

$$= 10\left(\cos 90^\circ + j\sin(90)\right) + 66 - j10$$

$$= \cancel{j10} + 66 - \cancel{j10} = 66$$

$$\frac{v(t)}{\sqrt{2}} = 66$$ $\varphi = 0$ $r = 66$

$$\boxed{v(t) = 66\sqrt{2}\cos(10^4 t)\text{ V}}$$

c) $\hat{I} = (15 + j5 + 10\angle 180^\circ)\, mA$

$\omega = 1k\ rad/s$

$\frac{i(t)}{\sqrt{2}} = 15 + j5 + 10(\cos 180^\circ + j\sin 180)\, m$

$= 15 + j5 - 10$

$\frac{i(t)}{\sqrt{2}} = 5 + j5$

$r = \sqrt{50} = 5\sqrt{2}$

$\theta = \tan^{-1}(1) = 45$

$\frac{i(t)}{\sqrt{2}} = 5\sqrt{2}\cos(10^3 t + 45^\circ)$

$\boxed{i(t) = 10\cos(10^3 t + 45^\circ)\, m}$

12.4 $\omega = 200\ rad/s$

a) $\hat{V}_1 = \frac{10 + j10}{2 - j3} \times \frac{2 + j3}{2 + j3}$

$= \frac{20 + 30j + 20j - 30}{4 + 9}$

$\hat{V}_1 = \frac{-10}{13} + \frac{50j}{13}$

$r = \sqrt{\frac{100 + 2500}{169}} = \sqrt{\frac{200}{13}} = \frac{10\sqrt{2}}{\sqrt{13}}$

$\varphi = \tan^{-1}\frac{50}{-10} + 180^\circ = 101.309^\circ$

$\frac{V_1(t)}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{13}}\cos(200t + 101.309^\circ)$

$V_1(t) = \frac{20}{\sqrt{13}}\cos(200t + 101.31^\circ)$

b) $\hat{V}_2 = (3 - 8j)(5e^{-j60^\circ})\, V$

$= (3 - 8j)(5(\cos(-60) - j\sin(60)))$

$= (3 - 8j)(\frac{5}{2} - \frac{5\sqrt{3}}{2}j)$

$= \frac{15}{2} - \frac{15\sqrt{3}}{2}j - 20j - 20\sqrt{3}$

$= \frac{15 - 40\sqrt{3}}{2} - j\frac{15\sqrt{3} + 40}{2}$

$-27.141 \quad 32.99038$

$r = 5\sqrt{73}$

$\varphi = 50.55 + 180 =$

$v_2(t) = 5\sqrt{146}\cos(200t + 50.55^\circ)$

$\frac{V_2(t)}{\sqrt{2}} = \sqrt{73}\, e^{-69.44}\, 5e^{-j60}$

$= 5\sqrt{146}\, e^{-129.44}$

$v_2(t) = 5\sqrt{146}\cos(200t - 129.44)\, V$

c) $\hat{I}_1 = \frac{10}{1 + j3} \times \frac{1 - j3}{1 - j3}$

$= \frac{10 - 30j}{1 + 9} = 1 - 3j$

$\frac{i_1(t)}{\sqrt{2}} = \sqrt{10}\, e^{-71.56}$

$i_1(t) = \sqrt{20}\cos(200t - 71.56^\circ)$

$\hat{I}_2 = \frac{1+3j}{1-j3}$ A

$\frac{i_2(t)}{\sqrt{2}} = \frac{\sqrt{10}\, e^{71.56j}}{\sqrt{10}\, e^{-71.56j}} = 1\, e^{143.13j}$

$\boxed{i_2(t) = \sqrt{2} \cos(200t + 143.13°) A}$

12.5

$v_1(t) = 50 \cos(\omega t - 45°)$

$v_2(t) = 25 \sin(\omega t)$

$v_3(t) = -[v_1(t) + v_2(t)]$

$= -50e^{-45j} - 25e^{-90j}$

$= 50(\cos(-45) - j\sin(45)) - 25(\cos(-90) + j\sin(-90))$

$= \frac{-50}{\sqrt{2}} + \frac{50}{\sqrt{2}} j + 25j$

$= -35.35 + 60.355j$

$r = 69.948$ $\qquad \varphi = \tan^{-1}\frac{60.355}{-35.35} + 180$

$\varphi = 120.36°$

$\boxed{v_3(t) = 69.948 \cos(\omega t + 120.36°)}$

12.6 $\hat{V}_1 = 5e^{j0}$, $\hat{V}_2 = 10e^{-135j}$

$\frac{v_1(t)}{\sqrt{2}} = 5(\cos(0°) + j\sin(0°))$

$\boxed{v_1(t) = 5\sqrt{2}\ V}$

$r = 5\sqrt{2}$

$\varphi = 0$

$v_1(t) = 5\sqrt{2} \cos(\omega t)$

$\frac{v_2(t)}{\sqrt{2}} = 10(\cos 135 - j\sin 135)$

$v_2(t) = 10$

$= -10 - 10j$

$r = 10\sqrt{2}$

$\varphi = 45 - 180 = -135$

$v_2(t) = 10\sqrt{2} \cos(\omega t - 135)$

$v_3(t) = 5\sqrt{2} - 10 - 10j$

$v_3(t) = -2.928 - 10j$

$v_3(t) = 10.42$

$\varphi = 253.67$

$v_3(t) = 10.42 \cos(\omega t + 253.67)$

12.7

$\hat{V}_1 = (2 + 6j)$ V

$\frac{v_1(t)}{\sqrt{2}} = \sqrt{40}\, e^{71.56j}$

$\hat{V}_1 = 6.32\, e^{(71.56 - 60°)j}$

$\hat{V}_1 = 6.32 e^{11.565j}$

~~$\hat{V}_1 = 6.32 \cos(\omega t + 11.565)$ V~~

$\hat{V}_1 = 6.32(\cos(11.565) + j\sin(11.565))$

$= 6.32(0.979 + j\,0.2004)$

$\boxed{\hat{V}_1 = 6.18728 + 1.26j}$

12.8 $\hat{V}_1 = -3 + j4$

$V_{2m} = 10$ V

$v_1(t) = -4.2426 + 5.656j$

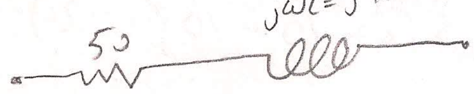
$\varphi_1 = -53.13 + 180 = 126.86°$

$\boxed{\varphi_2 = 216.869°}$

$\boxed{v_2(t) = 10 \cos(\omega t + 216.86°)}$

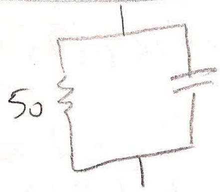
12.9

a)



$$\boxed{Z_{in} = 50 + j40\,\Omega}$$

b)



$$\frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{200 \cdot 200\, 10^3 \cdot 10^{-9}}$$

$$= \frac{-j}{4\times10^{-2}} = \frac{-j}{0.04}$$

$$Z_{in} = \frac{\frac{-50j}{0.04}}{50 - \frac{j100}{\cancel{0.04}}} = \frac{\frac{-5000j}{4}}{\frac{200 - j100}{4}}$$

$$Z_{in} = \frac{-5000j}{200 - j100} \times \frac{200 + j100}{200 + j100}$$

$$= \frac{-50j}{2-j} \times \frac{2+j}{2+j}$$

$$= \frac{-100j - 50j^2}{4+1} = \frac{50 - 100j}{5}$$

$$\boxed{Z_{in} = 10 - 20j}$$

c)

$$j\omega L = j\,2\times10^3\, 100\times10^{-3} = 200j$$

$$\frac{-j}{\omega C} = \frac{-j}{2\times10^3\times10^{-6}} = \frac{-j\,10^3}{2}$$

$$Z_{in} = 200j - j\frac{10^3}{2} = j\left(\frac{400 - 1000}{2}\right)$$

$$\boxed{Z_{in} = -300j}$$

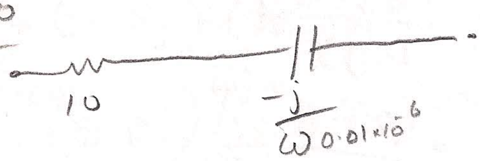
$$v_1(t) - v_2(t) = 15.134 + 28.952j$$

$$\angle V_3(t) = 62.40 \qquad V_{3m} = 32.67$$

$$v_3(t) = 32.67\cos(\omega t + 62.40)$$

$$\boxed{V_3(0) = 15.13V}$$

12.10



$$Z_{in} = 10 - \frac{j}{\omega\, 0.01\times10^{-6}}$$

$$\varphi = 12.5^\circ$$

$$\varphi = \tan^{-1}\left(\frac{-1}{\omega\, 0.1\times10^{-6}}\right)$$

$$\tan 12.5^\circ = \frac{-1}{\omega(0.1\times10^{-6})}$$

$$\omega = -4.5\times10^7 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi}$$

$$f = -7.18\times10^6\ Hz$$

~~12.11~~

$$\hat{V}_1 = 20\angle10^\circ\ V \qquad \hat{V}_2 = (9 - j17)\ V$$

$$r_1 = 20 \qquad r_2 = 19.23$$

a) $\hat{V}_1$ has greater mag.

b) $v_1(t) - v_2(t)$

$$\frac{v_1(t)}{\sqrt{2}} = 20(\cos 10 + j\sin 10)$$

$$= 19.69 + j3.47$$

$$\boxed{V_1(t) = 27.84 + j4.911}$$

$$v_2(t) = \cancel{9 - j17} = 12.72 - j24.041$$

$$\cancel{V_3(t) = -9 + j17 + 27.84 + j4.911}$$

$$\cancel{V_3(t) = -18.845 - j21.911}$$

$$= \cancel{6.78}\ \cancel{18.84 + 21.911j}$$

$\Rightarrow V_{3m} = 28.8902V$

$\varphi_3 = 49.302^\circ$

$V_3(0) = 28.8902\cos(49.302)$

$V_3(0) = 19.497$

$\hat{V}_2 = 9 - j17$

$\hat{V}_1 = 19.98 + 3.47j$

$\hat{V}_3 = \hat{V}_1 - V_2$
$= 10.98 + 20.47j$

$v_3(t) = 15.528 + 28.94j$

$V_{3m} = 32.8427$

$\varphi_3 = 61.7838^\circ$

$V_3(t) = 32.8427\cos(\omega t + 61.7838^\circ)$

$\boxed{V_3(0) = 15.52V}$

$\cancel{V_1(t) = 28.255 + 4.90j}$

$\cancel{r = 28.677 \quad \varphi = 9.838^\circ}$

$\cancel{V_1(t) = 28.677\cos(\omega t + 9.838)}$

$\cancel{0 = \cos\omega t\cos 9.838 - \sin\omega t}$

$\cancel{\cos(314t + 9.838) = 0}$

$V_1(t) = 27.84 + j4.911$

$r = 28.26 \qquad \varphi = 10$

$0 = 28.26(\cos 100\pi t + 10)$

$\cancel{\cos}\ 100\pi t = \cancel{\cos}\ 80$

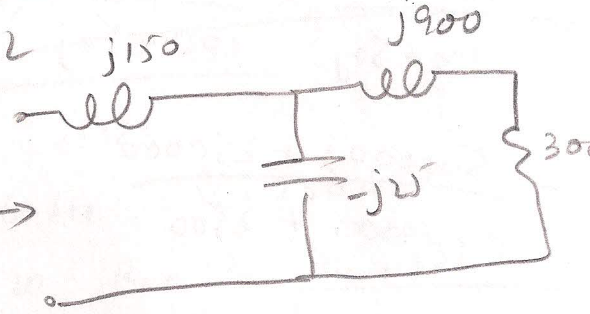
$t = \frac{80}{100\pi}$

$V_1 = 20\sqrt{2}\cos(314t + 10^\circ)$

$314t = 90 - 10$

$t = \frac{80}{314} = 0.25$

12.12



$$Z = \frac{(300 + j900)\cdot -j25}{(300 + j900) - j25} + j150$$

$$Z = \frac{-7500j + 22500}{300 + 875j} + j150$$

$$Z = \frac{-7500j + 22500 + 45000j - 131250}{300 + 875j}$$

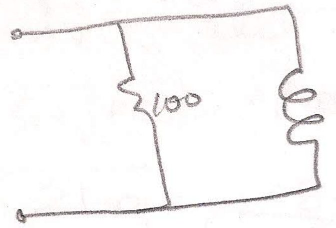
$$Z = \frac{37500j - 108750}{300 + 875j} \times \frac{300 - 875j}{300 - 875j}$$

$$= \frac{11250000j - 32625000 + 32812500 + 95156250j}{90000 + 765625}$$

$$= \frac{106406250j + 187500}{855625}$$

$$\boxed{Z = 0.22 + 124.36j}$$

12.13

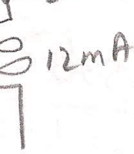


$$Z = \frac{5000j}{100+50j} \times \frac{100-50j}{100-50j}$$

$$Z = \frac{500000j + 250000}{10000 + 2500}$$

$$\boxed{Z = 40j + 20}$$

12.14 $i_L(t) = 20\cos(10^6 t)\ mA$



$\omega = 10^6$

$Z = j\omega L = j10^6\, 12mH = j12k\Omega$

$V_{(t)} = i_L(t)\, Z$

$= j12\times10^3\; 20\cos(10^6 t)\; 10^{-3}$

$\hat{V} = j\frac{240}{\sqrt{2}}\, e^{j(t)}$

$$V(t) = L\frac{d}{dt}\, 20\cos(10^6 t)\, m$$

$= 12\cdot10^{-3}\cdot 20\left(-\sin(10^6 t)\right)10^6$

$= -240\, e^{-90j}$

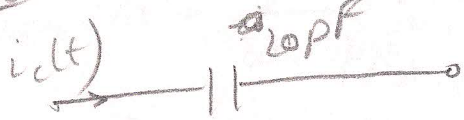
$= -240\left(\cos(-90) + \sin(-90)j\right)$

$V(t) = 240j$

$$\boxed{\hat{V} = \frac{240}{\sqrt{2}}j}$$

c) $\boxed{V_L(t) = -240\sin(10^6 t)\ V}$

12.15 $C = 20pF$



a) $Z_c = \frac{-j}{\omega C} = \frac{-j}{10^6 \cdot 20\cdot10^{-12}} = -j\frac{10^6}{20}$

$\boxed{Z_c = -j50k\Omega}$

b) ~~$V_c(t) = r$~~

~~$V_c(t) = -j50k\; 0.3m\cos(10^6 t)$~~

$i_c(t) = C\frac{dv(t)}{dt}$

$v(t) = \frac{1}{C}\int i_c(t)\,dt$

$= \frac{10^{12}}{20}\int 0.3\cos(10^6 t)\,10^{-3}\,dt$

$= \frac{10^{-3}\cdot0.3\times10^6}{20}\int 10^6\cos(10^6 t)\,dt$

$= \frac{10^3\cdot0.3}{20}\sin(10^6 t)$

$= 15\sin(10^6 t)$ (c)

$= 15\, e^{-90j}$

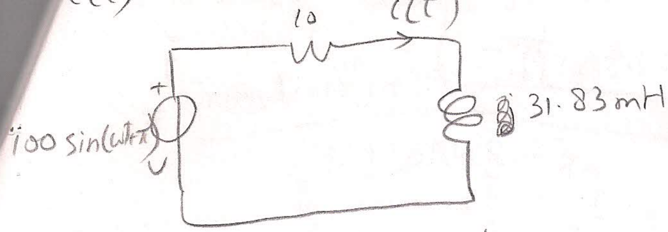
$= 15\left(\cos(-90) - j\sin 90\right)$

$V(t) = -15j$

$$\boxed{\hat{V} = \frac{-15}{\sqrt{2}}j}$$

$f = 50\,Hz \qquad e(t) = 100\sin(\omega t + \pi)\,V$

$i(t)$



$i(t) = \cancel{(j31.83m + 10)/100\sin(\omega t+\pi)}$

$\cancel{= 10\angle 72.55 / 100\angle 180-90}$ 0.1823

$\cancel{= 1000 \angle 72.55+90}$

$\cancel{= 1000\angle 162.558}$

$i(\cancel{t}) =$

$j\omega L = 100\pi \times 31.83m\, j = j9994.62 \times 10^{-3}$

$= \cancel{9.994}j\ 3.183\pi j$

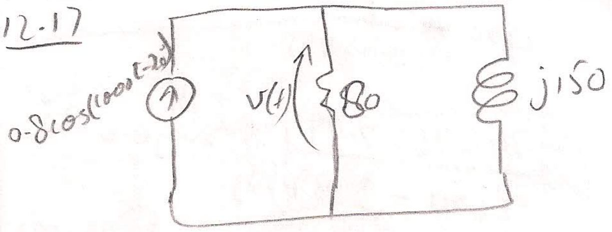
$$i(t) = \frac{100\sin(\omega t + \pi)}{10 + 9.994j\ \cancel{3.183\pi j}}$$

$$= \frac{100\, e^{(\pi - 90)j}}{14.13\, e^{44.98j}}$$

$$= 7.077\, e^{45j}$$

$$\boxed{i(t) = 7.077\cos(\omega t + 45^\circ)\ A}$$

12.17



$$Z_{eq} = \frac{12000j}{80 + 150j} \times \frac{80 - 150j}{80 - 150j}$$

$$Z_{eq} = \frac{960000j + 1800000}{28900}$$

$$\boxed{Z_{eq} = 62.283 + 33.218j}$$

$v(t) = 0.8\cos(1000t - 20^\circ)(62.883 + 33.218j)$

$= 0.8\, e^{-20j} \cdot 70.587\, e^{28.073j}$

$= 56.469\, e^{8.072j}$

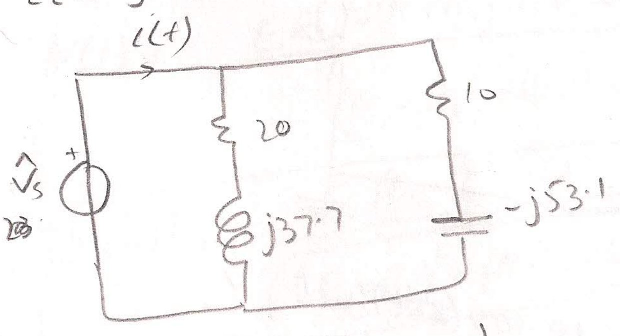
$$\boxed{v(t) = 56.469\cos(1000t + 8.072^\circ)\ V}$$

12.18 $\quad i(t) = ?$

$f = 60\,Hz \qquad \hat{V}_s = 230\angle 0^\circ V$

$R_1 = 20 \qquad R_2 = 10 \qquad Z_L = j37.7$

$Z_C = -j53.1$



$v_s(t) = 230\sqrt{2}\cos(120\pi t)$

$v_s(t) = 325.26\cos(376.8t)$

$$Z_{eq} = \frac{(10 - j53.1)(20 + j37.7)}{10 - j53.1 + 20 + j37.7}$$

$$Z_{eq} = \frac{200 + 377j - 1062j + 2001.87}{30 - j15.4}$$

$$Z_{eq} = \frac{2201.87 - 685j}{30 - 15.4j} \times \frac{30 + 15.4j}{30 + 15.4j}$$

$$= \frac{66056.1 - 20550j + 33908.79j + 10549}{1137.16}$$

$$= \frac{76605.1 + 13358.79j}{1137.16}$$

$$Z_{eq} = 67.365 + 11.747j$$

$$i(t) = 325.26\angle 0^\circ \,/\, (67.365 + 11.747j)$$

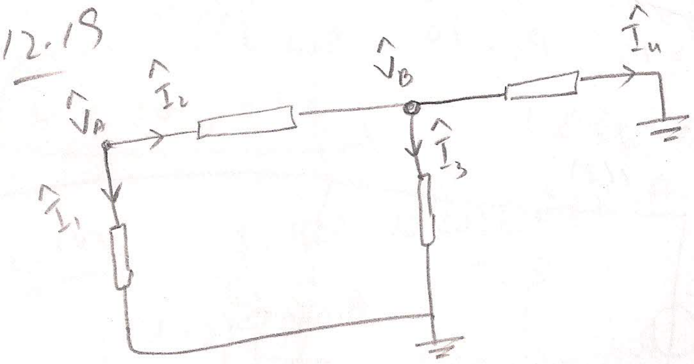
$$= 325.26\angle 0^\circ \,/\, 68.38\angle 9.89$$

$$= \cancel{22241.27\angle 9.89}$$

$$= 4.75\angle -9.89$$

$$\boxed{i(t) = 4.75\cos(\overset{376.8}{\cancel{1000}}\,t - 9.89^\circ)\ A}$$

12.19



$$v_A(t) = 10\cos\cancel{(\omega t)}(\omega t)\ V$$

$$V_A(t) = 10\angle 0^\circ$$

$$\boxed{\hat{V}_A = \frac{10}{\sqrt{2}}\ V}$$

$$v_B(t) = 10\sin(\omega t)$$

$$= 10\, e^{-90j}$$

$$= 10\left(\cos(-90) - j\sin(90)\right)$$

$$\boxed{\hat{V}_B = -\frac{10}{\sqrt{2}}j}$$

I) $i_1(t) = \sqrt{2}\cos(\omega t + 135^\circ)\ A$

$$\hat{I}_1 = \frac{\sqrt{2}}{\sqrt{2}}\, e^{135j}$$

$$= \cos 135 + j\sin 135$$

$$\boxed{\hat{I}_1 = \left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)}$$

$$i_4(t) = \cos(\omega t)$$

$$\boxed{\hat{I}_4 = \frac{1}{\sqrt{2}}}$$

$$I_2 = -\hat{I}_1$$

$$\hat{I}_3 = \hat{I}_2 - \hat{I}_4$$

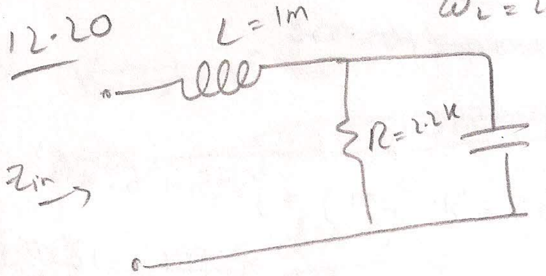
$$= \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}} - \cancel{\frac{1}{\sqrt{2}}}$$

$$\boxed{\hat{I}_3 = -j\frac{1}{\sqrt{2}}\ A}$$

$\omega_1 = 2\pi 10\ rad/s$

$\omega_2 = 2\pi 10^4\ rad/s$

12.20



$$\frac{-j}{\omega_1 C} = \frac{-j}{2\pi 10 \times 4.7\times 10^{-6}} = \frac{-j}{0.000295}$$

$$\frac{-j}{\omega_2 C} = \frac{-j}{2\pi 10^4 \times 4.7\times 10^{-6}} = \frac{-j}{0.29516}$$

$$j\omega_1 L = j\,2\pi 10 \times 10^{-3} = j\,0.0628$$

$$j\omega_2 L = j\,2\pi 10^4 \times 10^{-3} = j\,62.8$$

$$Z_{in}(\omega_1) = \frac{2.2\times 10^3\left(\frac{-j}{0.000295}\right)}{2200 - \frac{j}{0.000295}} + j\,0.0628$$

$$= \frac{-7457627.119j}{2200 - 3389.83j} + j\,0.0628$$

$$= \frac{-7457627.119j + j13.816 + 212.8813}{2200 - 3389.83j}$$

$$= \frac{-7457613.303j + 212.8813\ \cancel{2200}}{2200 - 3389.83}$$

$$Z_{in}(\omega_1) = \frac{-7457613.303j + 212.8013}{2200 - 3389.83j} \times \frac{2200+3389.83j}{2200+3389.83j}$$

$$= \frac{-1.64067\times10^{10}j + 2.528\times10^{18} + 468338.86 + 721631.41j}{16330947.43}$$

$$\boxed{Z_{in}(\omega_1) = 1.5\times10^{11} - j\,1.01\times10^{3}}$$

$$Z_{in}(\omega_2) = 62.8j + \frac{\frac{-2200j}{0.29516}}{2200 - \frac{j}{0.29516}}$$

$$= 62.8j + \frac{-2200j}{649.352 - j}$$

$$= \frac{40779.305j + 62.8 - 2200j}{649.352 - j}$$

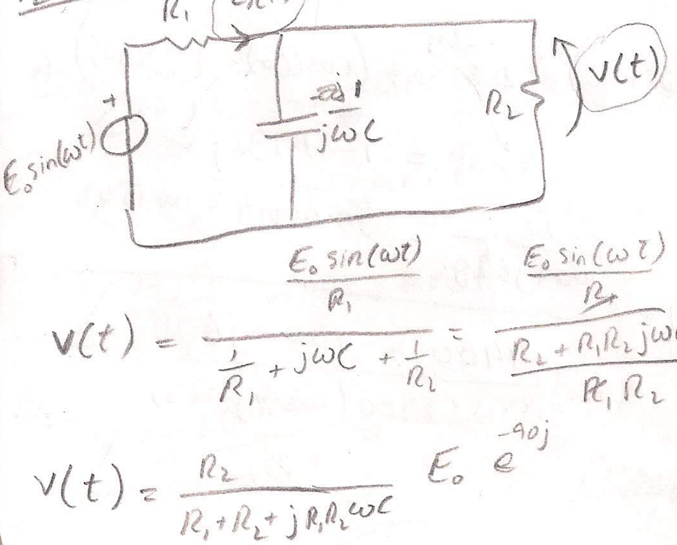
$$= \frac{62.8 + 38579.3056j}{649.352 - j} \times \frac{649.352+j}{649.352+j}$$

$$= \frac{25051549.25j - 38579.3056 + 40779.3056 + 62.8j}{421659.0199}$$

$$= \frac{2200 + 25051612.05j}{421659.0199}$$

$$\boxed{Z_{in}(\omega_2) = 0.0052 + 59.41j}$$

12.21



$$v(t) = \frac{\frac{E_0\sin(\omega t)}{R_1}}{\frac{1}{R_1} + j\omega C + \frac{1}{R_2}} = \frac{\frac{E_0\sin(\omega t)}{R_1}}{\frac{R_2 + R_1R_2 j\omega C + R_1}{R_1R_2}}$$

$$v(t) = \frac{R_2}{R_1+R_2+jR_1R_2\omega C} E_0 e^{-90j}$$

$$v(t) = \frac{E_0R_2}{R_1+R_2+jR_1R_2\omega C} \cdot -j$$

$$v(t) = \frac{-E_0R_2 j}{(R_1+R_2)+j(R_1R_2\omega C)} \times \frac{(R_1+R_2)-j(R_1R_2\omega C)}{(R_1+R_2)-j(R_1R_2\omega C)}$$

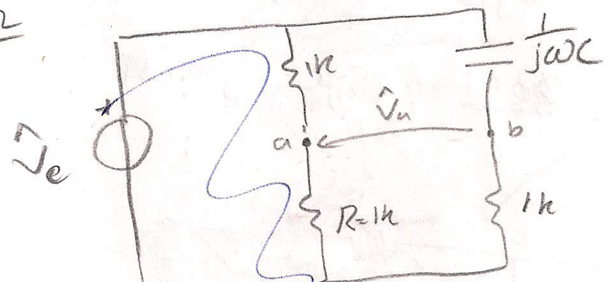
$$v(t) = \frac{-E_0R_2(R_1+R_2)j - E_0R_2^2R_1\omega C}{(R_1+R_2)^2 + (R_1R_2\omega C)^2}$$

$$|v(t)| = \sqrt{\frac{(E_0R_1R_2^2\omega C)^2}{[(R_1+R_2)^2+(R_1R_2\omega C)^2]^2} + \frac{(E_0R_2(R_1+R_2))^2}{[(R_1+R_2)^2+(R_1R_2\omega C)^2]^2}}$$

$$|v(t)| = \frac{E_0R_2\sqrt{(R_1+R_2)^2+(R_1R_2\omega C)^2}}{(R_1+R_2)^2+(R_1R_2\omega C)^2}$$

$$\boxed{|v(t)| = \frac{E_0R_2}{\sqrt{(R_1+R_2)^2+(R_1R_2\omega C)^2}}}$$

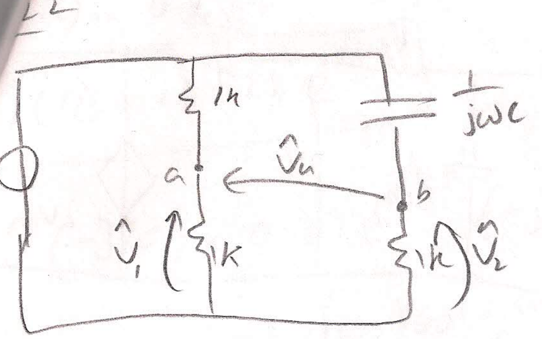
22



$$\arctan \frac{E_0R_2(R_1+R_2)}{E_0R_2^2R_1\omega C}$$

$$\arctan\left(\frac{R_1+R_2}{\omega R_1R_2C}\right)$$

12.22



$$\hat{V}_1 = \frac{\hat{V}_e}{2}$$

$$\hat{V}_2 = \frac{R\,\hat{V}_e}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{jR\omega C + 1}\hat{V}_e$$

$$\hat{V}_u = \hat{V}_1 - \hat{V}_2$$
$$= \hat{V}_e\left(\frac{1}{2} - \frac{j\omega RC}{j\omega RC + 1}\right)$$
$$= \hat{V}_e\left(\frac{j\omega RC + 1 - 2j\omega RC}{2(j\omega RC + 1)}\right)$$

$$\hat{V}_u = \left(\frac{1 - j\omega RC}{2(1 + j\omega RC)}\right)\hat{V}_e$$

$$2\frac{\hat{V}_u}{\hat{V}_e} = \frac{1 - j\omega RC}{\cancel{2}(1 + j\omega RC)}$$

since $\hat{V}_u$ lags $\hat{V}_e$ by $120°$

$$\angle\hat{V}_u - \angle\hat{V}_e = -120$$

As
$$\angle 1 - jx = -\arctan x$$
$$\angle 1 + jx = \arctan x$$
$$\angle\frac{1 - jx}{1 + jx} = -2\arctan x$$
$$\angle\frac{1 - j\omega RC}{1 + j\omega RC} = -2\arctan x$$

$$+2\arctan \omega RC = +\cancel{120}\ 60$$

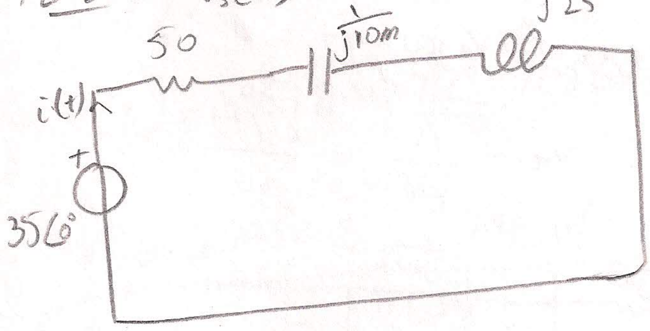
$$10^3 \times 62.8 \times 10^3\, C = 1.73$$

$$C = \frac{1.73}{628} \times 10^{-5}$$

$$C = 27.5 \times 10^{-9}\,F$$

$$\boxed{C = 27.5\,nF}$$

12.27 $\quad V_s(t) = 35\cos(1000t)\,V$



$$i(t) = \frac{35\,\cancel{\cos 0°} \angle 0°}{50 + \frac{1}{j10^{-2}} + j25}$$

$$= \frac{35\angle 0°}{50 - 10^2 j + 25j}$$

$$= \frac{35\angle 0°}{50 - 75j}$$

$$= \frac{35\angle 0°}{134.629\angle 68.19} \quad \frac{35}{90.138\angle -56}$$

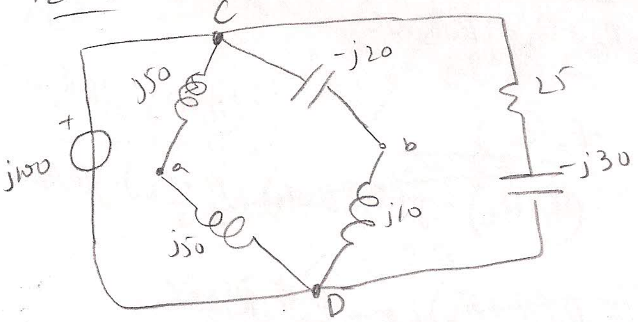
~~$i(t) = 0.25\,e$~~

$$\hat{I} = \frac{i(t)}{\sqrt{2}} = \frac{0.388}{\sqrt{2}}\angle 56.3$$

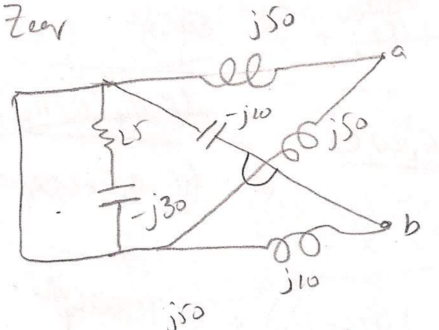
$$\boxed{\hat{I} = 0.27\,e^{j56.31°}}$$

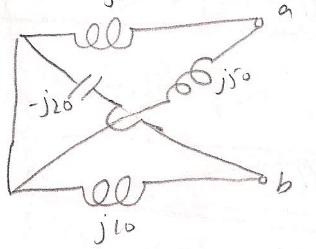
E

12-23



- Z_{eq}



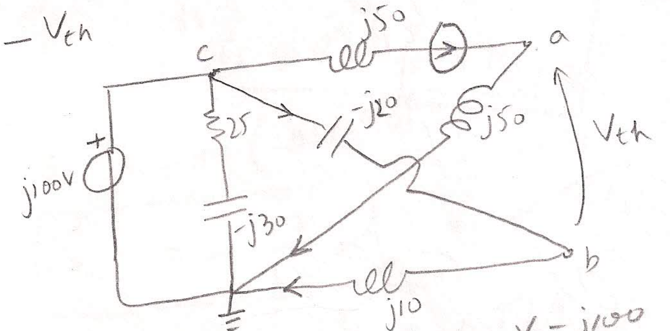


$$Z_{eq} = \frac{-j20 \times j10}{-j20 + j10} + \frac{j50 \times j50}{j50 + j50}$$

$$= \frac{\cancel{200}}{-j10} + \frac{-2500}{100j}$$

$$Z_{eq} = \cancel{20}j + 25j = j45\,\Omega$$

- V_{th}



$$-\frac{V_a - j100}{j50}$$

$V_c = j100$

$$-\frac{V_a - j100}{j50} = \frac{V_a}{j50}$$

$$V_a\left(-\frac{1}{j50} - \frac{1}{j50}\right) = -\frac{\cancel{100}\,2}{\cancel{50}}$$

$$V_a\left(\frac{-2}{j50}\right) = 2 \Rightarrow \boxed{V_a = +j50V}$$

$$+\frac{V_b - j100}{+j20} = \frac{V_b}{j10}$$

$$V_b\left(\frac{1}{j20} - \frac{1}{j10}\right) = 5\cancel{0}$$

$$V_b\left(\frac{1-2}{j20}\right) = 5$$

$$V_b = -100j$$

$$V_{th} = V_a - V_b$$

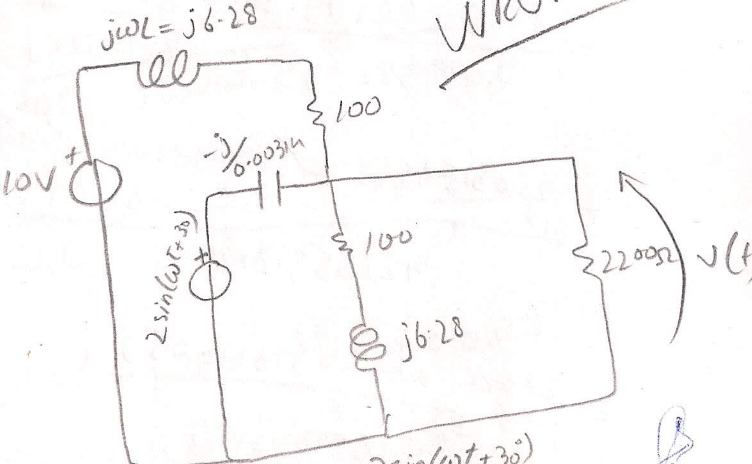
$$= +50j + 100j = 150j$$

12.24 $v(t)$? $E_0 = 10V$ $f = 1000Hz$

$e(t) = 2\sin(\omega t + 30)$ $L = 1mH$

$C = 0.5\mu F$ $R_1 = 100\Omega$ $R_2 = 2200\Omega$

WRONG



$$v(t) = \frac{\dfrac{10}{100 + j6.28} + \dfrac{2\sin(\omega t + 30°)}{\frac{1}{j0.00314}}}{\dfrac{1}{100 + j6.28} + \dfrac{1}{100 + j6.28} + \dfrac{1}{2200} + j0.00314}$$

$$2\sin(\omega t + 30°) = 2e^{-60j} = 2(\cos(60) - j\sin(60))$$

$$= 1 - 1.732j$$

$$v(t) = \frac{\dfrac{10}{100 + j6.28} + j0.00314 + 0.00543}{\dfrac{4400 + 100 + j6.28}{2200(100 + j6.28)}}$$

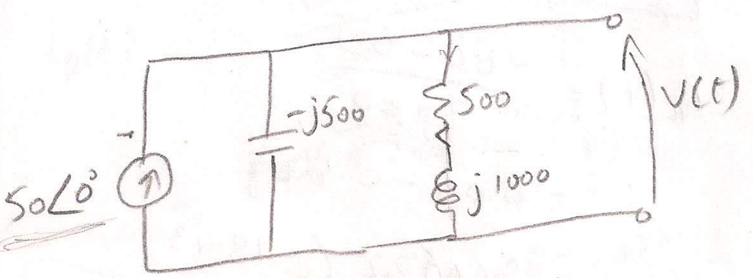
$\hat{i}(R) = 0.27\, e^{j56.31^\circ} \times 50$ (12.27)

$$\boxed{\hat{i}(R) = 13.728\, e^{j56.31^\circ}}$$

$$\hat{v}(-j10^2) = 0.27\, e^{j56.31}(-j10^2)$$
$$= 0.27(\cos 56.31 + j\sin 56.31)(-j10^2)$$
$$= (0.149 + j0.2246)(-100j)$$
$$= -14.9j + 22.46$$
$$= 26.95 \angle -33.56$$
$$\boxed{\hat{v}(C) = 26.95\, e^{-33.56j}}$$

$$\hat{v}(j25) = 0.27\, e^{j56.31}(j25)$$
$$= (0.149 + j0.2246)(j25)$$
$$= 3.725j - 5.615$$
$$\hat{v}(j25) = 6.738 \angle 146.43^\circ$$
$$\boxed{\hat{V}_L = 6.738\, e^{146.4j}}$$

12.28 $i_s(t) = 50\cos(2000t)\,mA$



$$Z_{eq} = \frac{(500 + j1000)(-j500)}{500 + j1000 - j500}$$

$$Z_{eq} = \frac{-j250000 + 500000}{500 + j500}$$

$$v(t) = \cancel{50}\left(\frac{500000 - j250000}{\frac{\cancel{500}}{10}(1+j)}\right) m$$

$$v(t) = \frac{50000 - j25000}{1+j} \times \frac{1-j}{1-j}\ m$$

$$v(t) = \frac{50000 - 50000j - j25000 - 25000}{2}\ m$$

$$v(t) = \frac{25000 - 75000j}{2}\ m$$

$$v(t) = 12500 - 37500j\ m$$

$$v(t) = 39528.4707 \angle -71.565\ m$$

$$\hat{v} = 27950\, e^{-71.565j}\ m\,V$$

$$\boxed{\hat{v} = 27.95\, e^{-71.565j}\ V}$$

c)

$i(R)$ $R=L$

$$i(t) = \frac{12.5 - 37.5j}{500 + j1000}$$

$$i(t) = \frac{(12.5 - 37.5j)(500 - j1000)}{250000 + 1000000}$$

$$= \frac{6250 - 12500j - 18750j - 37500}{1250000}$$

$$= \frac{-31250\ \cancel{25000} - 31250j}{1250000}$$

$$i(t) = -0.025 - 0.025j$$
$$= 0.035 \angle -135$$

$$\hat{I} = 0.025\, e^{-135j}$$

$$i(t) = 0.035\cos(2000t - 135^\circ)\,A$$

$$i_c(t) = \frac{12.5 - 37.5j}{-j500}$$

$$i_c(t) = \frac{(12.5 - 37.5j)\, j500}{250000}$$

$$= \frac{6250 - 18750j}{250000}$$

$$i_c(t) = 0.025 - 0.075j$$

$$i_c(t) = \cancel{0.00625}\; 0.07905 \angle -71.56$$

$$\hat{I}_c = \cancel{0.0044}\; 0.055\, e^{-j71.56}$$

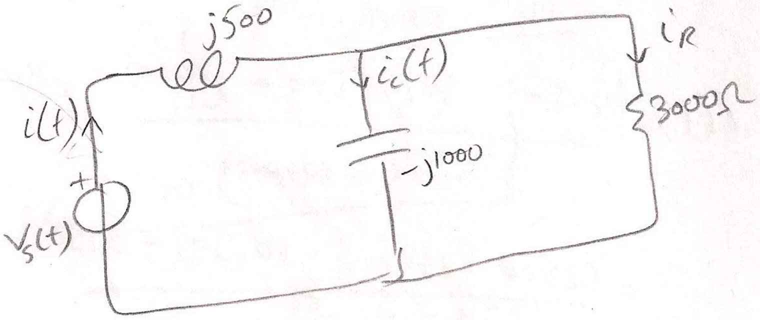
$$i_c(t) = 0.055\cos(2000t - 71.56)\ A$$

12.29 $\quad i(t),\ i_c(t),\ i_R(t)$

$v_s(t) = 100\cos(2000t)$ V

$L = 250\,mH \qquad C = 0.5\,\mu F$

$R = 3\,k\Omega$



$$R\|C = \frac{3000(-j1000)}{3000 - j1000} = \frac{-j3000}{3-j} \times \frac{3+j}{3+j}$$

$$R\|C = \frac{-9000j + 3000}{9+1} = \frac{-9000j + 3000}{10}$$

$$\boxed{R\|C = 300 - 900j}$$

$$V(R\|C)(t) = \frac{300-900j}{300-900j+j500} \times 100$$

$$= \frac{300-900j}{300-400j} \times 100$$

$$= \frac{300-900j}{3-4j} \times \frac{3+4j}{3+4j}$$

$$V(t) = \frac{900+3600+1200j-2700j}{9+16}$$

$$= \frac{4500 - 1500j}{25}$$

$$\boxed{V(t) = 180 - 60j} \leftarrow (R\|C)$$

$$i_c(t) = \frac{180-60j}{-j1000} \times \frac{j1000}{j1000}$$

$$= \frac{180000j + 60000}{(1000000)}$$

$$i_c(t) = 0.06 + 0.18j$$

$$= \cancel{0.3?4}$$

$$= 0.189 \angle 71.57^\circ$$

$$\boxed{i_c(t) = 0.189\cos(2000t + 71.57)}$$

$$i_R(t) = \frac{180 - 60j}{3000}$$

$$= 0.06 - 0.02j$$

$$= 0.0632 \angle -18.43$$

$$\boxed{i_R(t) = 0.0632\cos(2000t - 18.43^\circ)}$$

$i_C(t) = \frac{(300-900j)(}{(300-900j)+j500} \times V(t)$

$= \frac{(300-900j)\cancel{100}}{300-\cancel{400}j}$

$= \frac{(300-900j)(3+4j)}{9+16}$

$= \frac{900+1200j-2700j+3600}{25}$

$= \frac{4500-1500j}{25}$

$V_{CR}(t) = 180-j60$

$i_C(t) = \frac{180-j60}{-j1000}$

$= \frac{60+18j}{1000}$

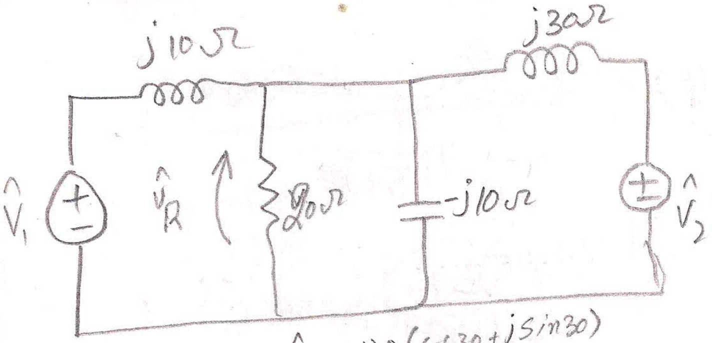
$= \frac{6}{100} + \frac{18}{1000}j = 0.06+0.018j$

$i_R(t) = \frac{180-j60}{3000}$

$= 0.06 - j.02$

$i_t = 0.06+0.18j+0.06-j0.02$

$= 0.12+0.16j$



$\hat{V}_2 = 120(\cos 30 + j\sin 30)$

$Z_L = j10\Omega$, $= 120$

$\hat{V}_R = \frac{\frac{-j100}{\cancel{j10}} + \frac{(103.92+j60)j}{j30}}{\frac{-\cancel{j}}{\cancel{j}10} + \frac{1}{20} + \frac{+\cancel{j}}{\cancel{+j}10} + \frac{-j}{j30}}$

$= \frac{-j10 + \frac{103.92+j60}{j30}}{\frac{\cancel{-6j}+3\cancel{j}+\cancel{6j}-2j}{60}}$

$= \frac{2(-j300 - j103.92+j60)}{3-2j}$

$= -2j(-j300 - j103.92+60)$

$= -600 - 207.84 - 120j$

$= -808.68 - 120j$

=

$$v(t) = \frac{50}{-\frac{1}{j500} + \frac{1}{5+j100}} \text{ mV}$$

$$= \frac{50(j500)(5+j100)}{-(5+j100)+j500} \text{ mV}$$

$$= \frac{2500j(5+j100)}{-5-j100+j500} \text{ mV}$$

$$= \frac{125j - 250000}{j400-5} \times \frac{j400+5}{j400+5}$$

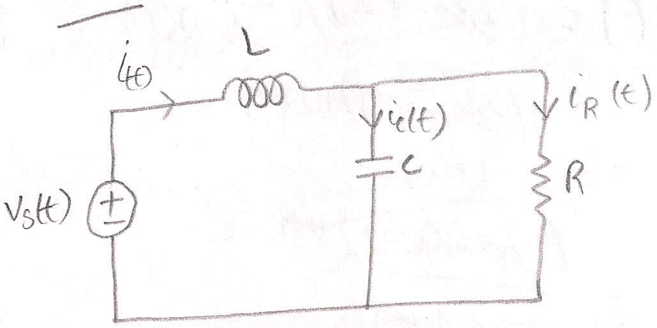
$$= \frac{-50000 + 625j - 100000000j - 1250000}{160000+25}$$

$$= \frac{-1300000 \quad -9999375}{160025}$$

$$= -8.12 - j62.48$$

$=$

12.29



$$V_s(t) = 100\cos(2000t) \text{ V}$$

$$V_s(t) = 100e^{j0} = 100\angle 0$$

$$\omega = 2000$$

$$Z_L = j2000 \times 250\times 10^{-3}$$

$$= j\,500\cancel{000}\ \Omega$$

$$Z_C = \frac{-j\times 10^6}{2000\times 0.5} = -j1000$$

$$R = 3000\ \Omega$$

$$Z_{eq} = Z_L + \frac{RZ_c}{R+Z_c}$$

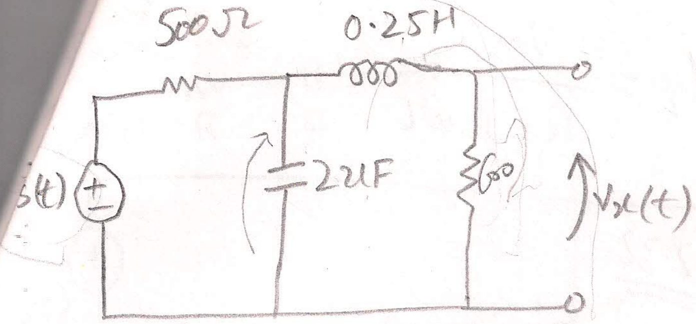
$$= R_L + \frac{3000\times -j1000}{3000+(-1000j)}$$

$$= Z_L + \frac{-3000{,}000j}{3000-1000j} \times \frac{3000+1000j}{3000+1000j}$$

$$= Z_L + \frac{-9000{,}000{,}000j + 3000{,}000{,}000}{9000{,}000 + 1000{,}000}$$

$$= j500 + \underbrace{300 - 900j}_{} \rightarrow \underline{\underline{R \| C}}$$

$$= 300 - 400j$$



$$Z_C = \frac{-j \times 10^6}{1000 \times 2} = -j500\,\Omega$$

$$Z_L = j1000 \times \frac{1}{4} = j250\,\Omega$$

$$V_x = \frac{\frac{5}{500}}{\frac{1}{500} + \frac{1}{-j500} + \frac{1}{600+j250}}$$

$$= \frac{5}{\frac{-j500+500}{-500 \cdot j500} + \frac{600-j250}{600^2+\cancel{j}250}} \text{ mV}$$

$$= \frac{5}{\frac{+\cancel{500}(j-1)}{+\cancel{500}\,j500} + \frac{600-j250}{360000+62500}}$$

$$= \frac{5}{\frac{j-1}{j500} + \frac{600-j250}{4225}}$$

$$= \frac{5}{\frac{j-1}{j500} + 0.142 - j.00591} \text{ mV}$$

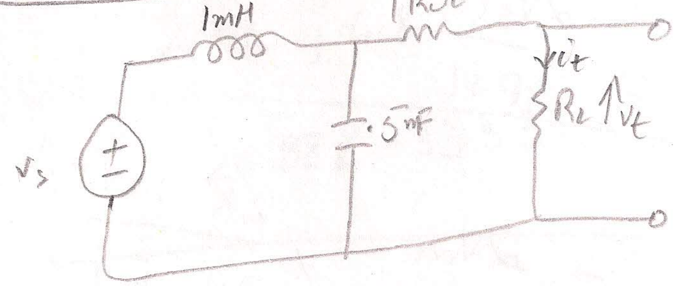
$$= \frac{5(j500)}{j-1+j71+j2.955} \text{ mV}$$

$$= \frac{2500j}{-1+j74.955} \text{ mV}$$

$$= \frac{(2500j)(-1-j74.955)}{5619.25}$$

$$= \frac{-2500j + 33.35}{5619.25}$$

$$= -0.44 + j33.35$$



$V_S = 30\angle 0$

$$X_L = j10^{-3} \times 10^6 = j1000\,\Omega$$

$$X_C = \frac{-j}{10^6 \times 10^{-9} \times .5} = -j\frac{10^3 \times 10}{5} = -j2000\,\Omega$$

$$Z_{eq} = 1000 + \frac{j1000 \times -j2000}{j1000 - j2000}$$

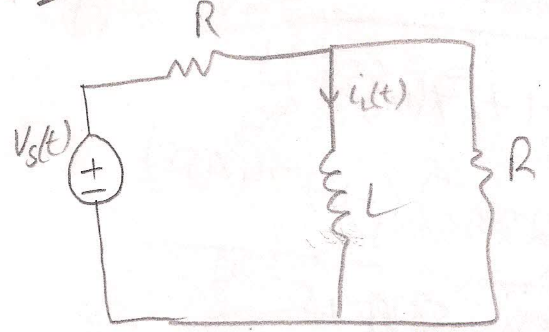
$$= 1000 + \frac{\cancel{j1000} \times -j2000}{\cancel{j1000}(1-\cancel{j2000})}$$

$$= 1000 + \frac{+j2000}{+1}$$

$$= 1000 + j2000$$

$\cancel{X_{R_L} = j10^6 \times}$

12·31



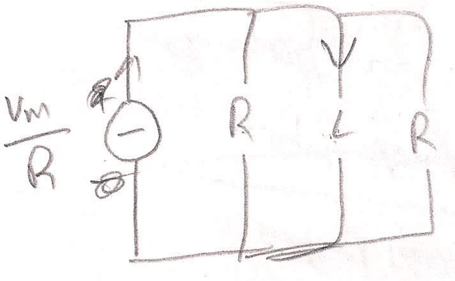
$V_s(t) =$

$$i_L(t) = \frac{2\cancel{R}}{2R+L} \times \frac{V_s(t)}{\cancel{R}}$$

$$= \frac{2V_s(t)}{2R+L}$$

$$= \frac{2V_m}{2R+j\omega L} \times \frac{2R-j\omega L}{2R-j\omega L}$$

$$= \frac{4RV_m - j2V_m\omega L}{4R^2+\omega^2 L^2}$$



$$\frac{\frac{\cancel{R}}{2}}{\frac{R}{2}+L} \times \frac{V_m}{\cancel{R}}$$

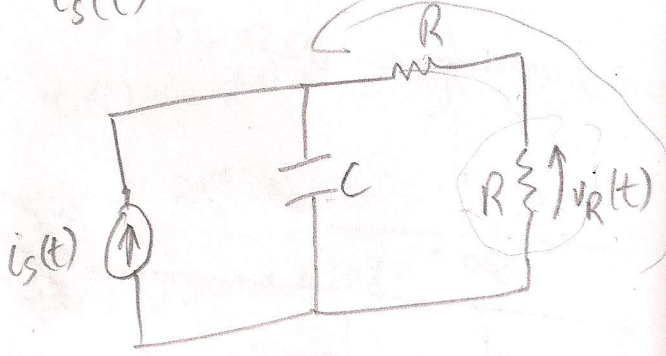
$$\frac{V_m}{R+2L}$$

$$i_t = \frac{V_m}{R+j2\omega L}$$

12·32

$i_s(t) = i_m \angle -90°$

$i_s(t) = i_m(-j) = -j i_m$



$$V_R = \frac{1}{2}\left(\frac{-j i_m}{\frac{1}{ZC}+\frac{1}{2R}}\right)$$

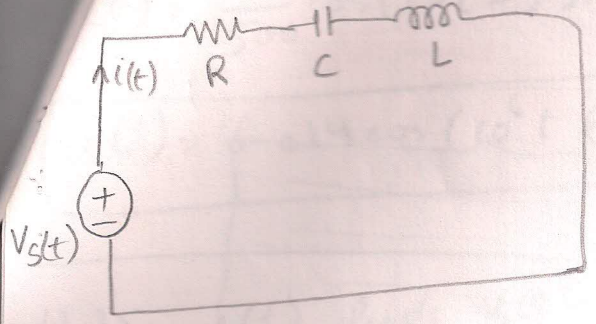
$$= \frac{1}{2}\,\frac{-j i_m}{\frac{2R+Z_C}{2RZ_C}}$$

$$= \frac{\frac{1}{\cancel{2}} \times \cancel{2}RZ_C \times (-j i_m)}{2R+Z_C}$$

$$= \frac{-\cancel{j}R(\cancel{j}\omega C) i_m}{2R+ZC}$$

$$= \frac{-R\frac{1}{\cancel{j\omega C}} i_m}{\frac{j2R\omega C+1}{\cancel{j\omega C}}} = \frac{-jR i_m}{1+j2R\omega C}$$

$-\tan^{-1} 2R\omega L$



$V_S = 35\cos(1000t)$

$= 35e^{0j}$

$= 35\,V$

$Z_L = j1000\times25\times10^{-3} = j25\,\Omega$

$Z_C = \dfrac{-j}{1000\times10\times10^{-6}} = -j10^{2}$

$= -j100\,\Omega$

$R = 50\Omega$

$i(t) = \dfrac{35}{50-j100+j25}\ A$

$= \dfrac{35}{50-j75}\times\dfrac{50+j75}{50+j75}$

$= \dfrac{1750+j2625}{2500+5625}$

$= 0.215 + j0.323$

$|i| = 0.388$

$\angle i = \tan^{-1}\dfrac{0.323}{.215}$

$= 56.35$

$\hat{i} = \dfrac{0.388\angle 56.35}{\sqrt{2}}$

$= 0.27\angle 56.35$

$\hat{V}(R) = 0.27\angle 56.31^{\circ}\times 50$

$= 13.728\angle 56.31$

$\hat{V}(-j100) = 0.27\angle 56.31^{\circ}\times(-j100)$

$= 0.27(\cos 56.31 + j\sin 56.31)(-j100)$

$= 0.27(0.55+j.83)(-j100)$

$= 0.27(-j55+83)$

$= 22.41 - j14.85$

$\hat{V}(C) = 26.88\angle -33.53$

$\hat{V}(L) = 0.27\angle 56.31\times j25$

$= 0.27(0.55+j.83)j25$

$= 0.27(j13.75 - 20.75)$

$= -5.605 + j3.7125$

$= 6.718\angle 146.469^{\circ}$

$$v(t) = \frac{300j - 125}{85j - 85} \times \frac{-85 - 85j}{-85 - 85j}$$

$$= \frac{-25500j + 10625j + 10625 + 25500}{14450}$$

$$= \frac{36125 - 14875j}{14450}$$

$$v(t) = 2.5 - 1.029j$$

$$v_x(t) = \frac{600}{600 + j250} \times (2.5 - 1.028j)$$

$$= \frac{1500 - 617.4j}{600 + j250} \times \frac{600 - j250}{600 - j250}$$

$$= \frac{-370440j - 375000j + 900000 - 154375}{422500}$$

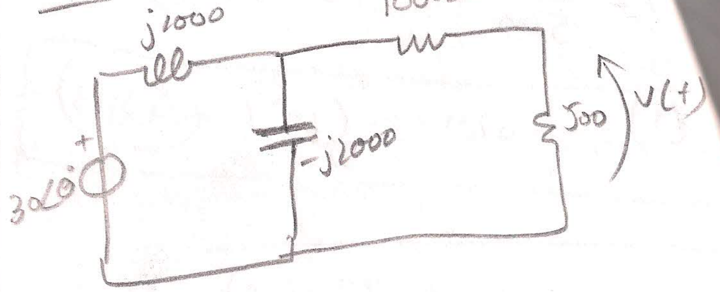
$$= \frac{745625 - 745440j}{422500}$$

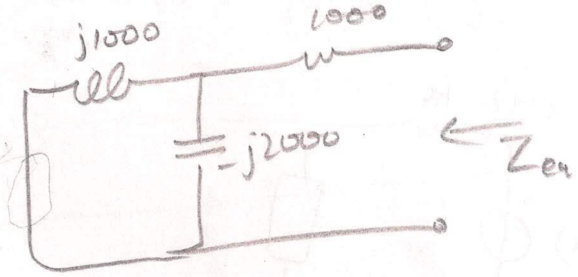
$$v_n(t) = 1.7647 - 1.764j$$

$$= 2.49 \angle -45^\circ$$

$$\boxed{v_n(t) = 2.49 \cos(1000t - 45^\circ)\ V}$$

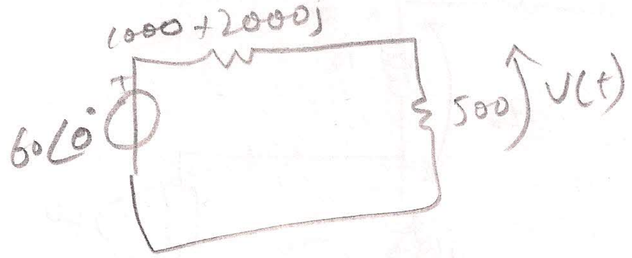
12.34





$$Z_{eq} = \frac{2 \times 10^{6}}{-j\cancel{1000}} = 2000j + 1000$$

$$V_{eq} = \frac{-j2000}{-j\cancel{1000}}\ 30 = 60\angle 0^\circ$$



$$v(t) = \frac{500}{1500 + 2000j} \cdot 60$$

$$= \frac{60}{3 + 4j} \times \frac{3 - 4j}{3 - 4j}$$

$$= \frac{180 - 240j}{9 + 16}$$

$$\cancel{v(t) = 3.529 - 14.1176j}$$

$$\cancel{v(t) = 14.55}$$

$$= 7.2 - 9.6j$$

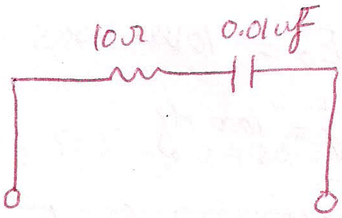
$$= 12 \angle -53.13$$

$$\boxed{v(t) = 12 \cos(10^6 t - 53.13)}$$

Solving STARS

12.10

$f = ?$



$\theta = 12.5^\circ = \angle V_\theta + \angle i$ { Current will Lead.

Solution

$$Z = 10 + \frac{-j \times 10^6}{2\pi f \times 0.01}$$

$$= \left(10 - j\frac{15915494.31}{f}\right) \Omega$$

$$\angle Z = \angle V - \angle i = -12.5^\circ$$

So

$$-12.5^\circ = \tan^{-1}\left(\frac{-\frac{15915494.31}{f}}{10}\right)$$

$$+0.22 = +\frac{15915494.31}{10f}$$

$$f = \frac{15915494.31}{2.2} = \boxed{7.23\ MHz}$$

12.24

Find $v(t)$,

$E_0 = 10V$

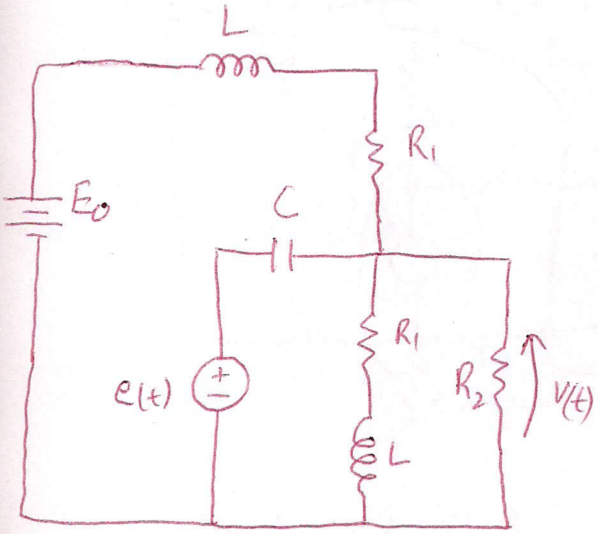
$f = 1000\ Hz$

$e(t) = 2\sin(\omega t + 30^\circ)$

$L = 1\ mH$

$C = 0.5\ uF$

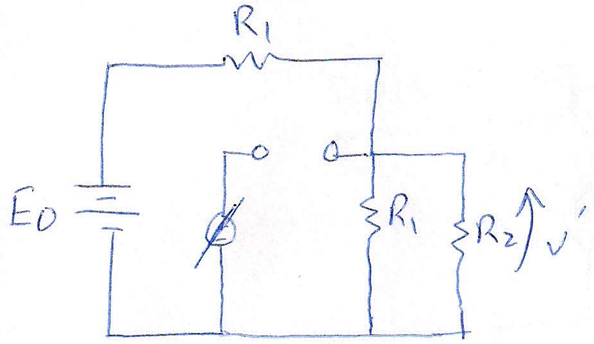
$R_1 = 100\ \Omega$, $R_2 = 2200\ \Omega$



Solution

Super-Position

i) $v(t)$ According to $E_0 = v'$



MILLMANN

$$v' = \frac{10/100}{\frac{1}{100} + \frac{1}{100} + \frac{1}{2200}}$$

$$v' = \frac{10}{2 + \frac{1}{2.2}} = \frac{22}{5.4} = \boxed{4.07V}$$

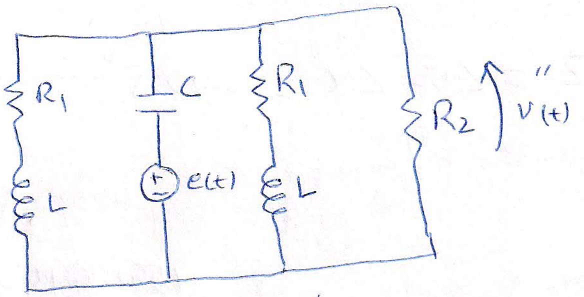
ii) v'' (According to $e(t)$)

$e(t) = 2\angle -60^\circ = (1 - \sqrt{3}j)\ v$

$Z_L = j62.83\ \Omega$

$Z_C = -j318\ \Omega$

This is the mistake. which changes the whole Result. Method is Right.



$$v''(t) = \frac{\frac{1-\sqrt{3}j}{-j318}}{\frac{2}{100+j62.83} + \frac{1}{j318} + \frac{1}{2200}}$$

$$= \frac{\frac{1-\sqrt{3}j}{\cancel{-j318}}}{\frac{2(-j318)(2200) - 2200(100+j62.83) + (100+j62.83)(-j318)}{(2200)(100+62.83j)\cancel{(-j318)}}}$$

$$= \frac{(1-\sqrt{3}j)(2200)(100+62.83j)}{-j1399200 - 220000 - j138226 - j31800 + 19980}$$

$$V(t)'' = \frac{220000 + j138226 - j381051 + 239414.455}{-200020 - j1569226}$$

$$= \frac{459414.545 - j242825}{-20020 - j1569226} \quad \text{PHASOR FORM} \Rightarrow \frac{519639.98 \angle -27.85^\circ}{1569353.7 \angle -90.73^\circ}$$

$\theta - 180 \Rightarrow -90.73^\circ$

$$V(t)'' = (0.33 \angle +62.88^\circ)\,V$$

#ERROR

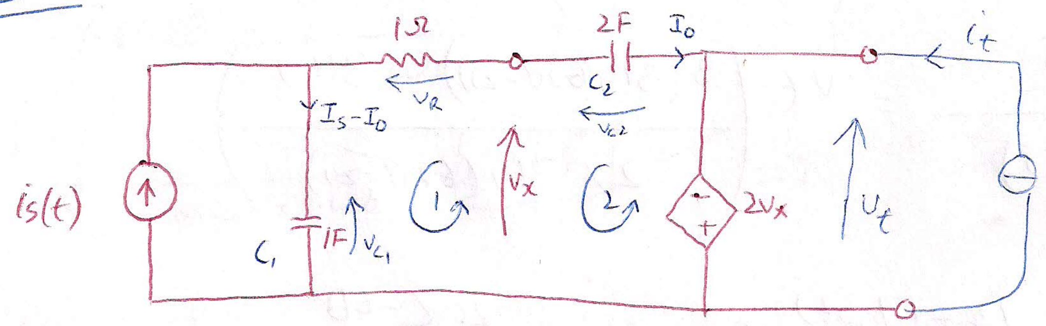
ANSWER NOT VERIFIED

SO FINALLY

$$V(t) = V' + V(t)''$$

$$= \left[4.07 + 0.33\cos(\omega t + 62.88^\circ)\right] V$$

12.26



$i_s(t) = 10\sin(\omega t + 30^\circ)$ A

$\omega = 314$ rad/s

Solution

$i_s(t) = (5 - 8.66j)$ A

$Z_{C_1} = -j/314\ \Omega$

$Z_{C_2} = \frac{-j}{628}\ \Omega$

$V_t + 2V_x = 0$

$V_t = -2V_x \Rightarrow V_x = -\frac{V_t}{2}$ —— (i)

KVL1

$V_x + I_o + \left(\frac{-j}{314}\right)(I_s - I_o) = 0$

$V_x + I_o + \left[\frac{j}{314} \times (5 - 8.66j - I_o)\right] = 0$

$I_o\left(1 - \frac{j}{314}\right) = -V_x - \left(\frac{8.66}{314} + \frac{5}{314}j\right)$

$$I_0 = \frac{-314V_x - 8.66 - 5j}{314 - j} \quad \text{———— (ii)}$$

KVL 2 $\quad I_0\left(\frac{-j}{628}\right) - V_x - 2V_x = 0$

Putting value from i, ii $\Rightarrow$

$$3\left(-\frac{V_t}{2}\right) = -\frac{j}{628}\left(\frac{-314\left(\frac{-V_t}{2}\right) - 8.66 - 5j}{314 - j}\right)$$

$$-\frac{3}{2}V_t = \frac{-\cancel{157}jV_t}{2\cancel{314}(628-2j)} + \frac{8.66j}{314(628-2j)} - \frac{5}{314(628-2j)}$$

$$\frac{5}{314(628-2j)} - \frac{8.66j}{314(628-2j)} = V_t\left(\frac{3}{2} - \frac{157j}{314(628-2j)}\right)$$

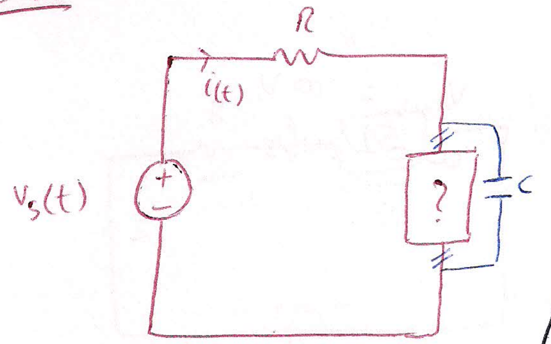
$$\frac{5 - 8.66j}{\cancel{314(628-2j)}} = V_t\left(\frac{3(314(628-2j)) - 314j}{2 \times \cancel{314(628-2j)}}\right)$$

$$V_t = \frac{10 - 17.32j}{591576 - j\,2198} = \frac{20\angle -60}{591580\angle -0.21}$$

$$= 3.38 \times 10^{-5} \angle -59.8^\circ$$

$$\boxed{\begin{aligned} V(t) = V_{eq} &= \left[3.38 \times 10^{-5} \cos(\omega t - 59.8^\circ)\right] V \\ Z_{eq} &= 0\,\Omega \end{aligned}}$$

12.35



$\angle i_s - \angle V = 55^\circ \Rightarrow$

$V_s(t) = 120\sqrt{2}\cos(120\pi t)\,V$

$R = 10\,\Omega$

(My Answer was Right)

Solution

i) As current is leading the phase of voltage so the unknown ELEMENT is a capacitor.

ii) $C = ?$

$$Z_{eq} = 10 + \frac{1}{j\omega C}$$

$$\angle Z_{eq} = -55^\circ = \tan^{-1}\left(\frac{-\frac{1}{\omega C}}{10}\right)$$

$$+1.428 = +\frac{1}{10\omega C} \qquad \left\{ \omega = 120\pi \text{ rad/s} \right\}$$

$$C = \frac{1}{10 \times 120\pi \times 1.428} = \boxed{186\,\mu F} \checkmark \Rightarrow Z_C = \frac{-j}{\omega C} = \frac{-j10^6}{186 \times 120\pi} = -j14.26\,\Omega$$

ii) $i(t)_p = ?$ $\equiv$ Peak value is $\hat{V}(t) = V_p e^{j\theta}$

$\Downarrow$

The magnitude of the phasor.

So

$$i(t) = \frac{120\sqrt{2}\angle 0^\circ}{R + Z_C} = \frac{120\sqrt{2}\angle 0^\circ}{10 - j14.26}$$

$$i(t) = (9.74\angle 54.96^\circ)\,A$$

$$= 9.74e^{j54.96^\circ}\,A \Rightarrow$$

So $I_p = 9.74\,A$

But Given Answer is: $\frac{9.74}{\sqrt{2}} = 6.88A$

?

if we Assume
$V_s(t) = 120\cos(120\pi t)\,V$
As Result
$I_p = 6.88\,A$
VERIFIED

CH # 12

Rectangular Form: $\hat{x} = a \mp jb$

Magnitude $= \sqrt{a^2+b^2} = |\hat{x}|$

Phase Angle $= \angle\hat{x} = \tan^{-1}\frac{b}{a}$

if $\hat{x} = a+jb$, $\tan^{-1}\frac{b}{a} = \theta^\circ$

$\hat{x} = a-jb$, $\tan^{-1}\frac{-b}{a} = \theta^\circ$

$\hat{x} = -a+jb$, $\left(\tan^{-1}\frac{b}{-a}\right)+180^\circ = \theta^\circ$

$\hat{x} = -a-jb$, $\left(\tan^{-1}\frac{b}{a}\right)-180^\circ = \theta^\circ$

Phasor Form $\hat{x} = |\hat{x}|\angle\theta^\circ$

$\hat{x} = |\hat{x}|e^{j\theta}$ (Polar Form)

Sinusoidal Form $= |\hat{x}|\cos(wt \mp \theta^\circ) = \hat{x}$ (i)

w = Angular Frequency $= 2\pi f_o$

$\angle = \theta^\circ$

$\hat{x} = |\hat{x}|\sin(wt \mp \theta^\circ)$ (ii)

$\angle = \theta^\circ - 90^\circ$

CH # 13

If we use R.M.S values OR Phasor values

Active Power $P = |\hat{V}||I|\cos\theta$ W , $P = \frac{1}{2}|\hat{V}||\hat{I}|\cos\theta$ w

Re........ $Q = |V||I|\sin\theta$ KVAR , $Q = \frac{1}{2}|\hat{V}||\hat{I}|\sin\theta$ VAR

Apparent Power $A = |V||I|$ VA , $A = \frac{1}{2}|\hat{V}||\hat{I}|$ VA

Complex Power $S = VI^*$, $S = \frac{1}{2}\hat{V}\hat{I}^*$ VA

$= P+jQ$ VA , $= Ae^{j\theta}$

$= Ae^{j\theta}$, $= P+j\theta$

$\angle S = \angle Z = \angle v - \angle i$, $\angle S = \angle_Z = \angle\hat{v} - \angle\hat{i}$

$$X_{rms} = \frac{\hat{X}}{\sqrt{2}}$$

$$\theta = \theta_v - \theta_i$$

$v(t) = 168\cos(377t + 45^\circ)$ V

$i(t) = 0.88\cos(377t)$ A

$P = V_{rms} \cdot I_{rms} \cos(\varphi_v - \varphi_i)$ ✓

$= \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \cos(45 - 0)$

$P = 73.92(0.707)$

$\boxed{P = 52.269\,W}$

Since it is +ve so element absorbs energy.

$Q = V_{rms} \cdot I_{rm} \sin(\varphi_v - \varphi_i)$ ✓

$= \frac{168 \times 0.88}{2} \sin(45 - 0)$

$= 73.92(0.707)$

$\boxed{Q = 52.27\,VAR}$

b) $v(t) = 285\cos(2500t - 68)$ V

$i(t) = 0.66\cos(2500t)$ A

$P = V_{rms} \cdot I_{rm} \cos(-68 - 0)$

$P = \frac{285 \cdot 0.66}{2} \cos(-68)$

$= 94.08(0.374)$

$\boxed{P = 35.24\,W}$

Absorbs energy

$Q = V_{rms}\, I_{rm} \sin(-68 - 0)$

$= 94.08(-0.927)$

$\boxed{Q = -87.229\,VAR}$

c) $v(t) = 168\cos(377t + 45^\circ)$ V

$i(t) = 0.88\cos(377t - 60^\circ)$ A

$P = \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \cos(45 + 60)$

$= 73.92\cos(105)$

$\boxed{P = -19.13\,W}$

$-ve \Rightarrow$ supplies energy

$Q = \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \sin(105)$

$= 73.92(0.965)$

$\boxed{Q = 71.401\,VAR}$

d) $v(t) = 285\cos(2500t - 68)$ V

$i(t) = 0.8\sin(2500t)$ A

$P = \frac{285}{\sqrt{2}} \times \frac{0.88}{\sqrt{2}} \cos(-68 + 90)$

$= \cancel{125.4(0.3746)}$

$\boxed{\cancel{P = 42.70\,W}}$

$\boxed{P = 116.268\,W}$

absorbs energy

$Q = \frac{285}{\sqrt{2}} \times \frac{0.88}{\sqrt{2}} \sin(-68 + 90)$

$= 125.4(0.374)$

$\boxed{Q = 46.97\,VAR}$

(REMEMBER Thes QUESTIONS MAY vary, But METhod is SAME)

13.2

a) $\hat{V} = 120\angle 30^\circ$ V $\quad \hat{I} = 20\angle 75^\circ$

$$Z = \frac{120\angle 30^\circ}{20\angle 75} = 6\angle -45\,\Omega$$

$$\boxed{Z = 6e^{-j45}\,\Omega}$$

b) $A = 3.3$ kVA $\quad Q = -1.8$ kVAR

$|\hat{I}| = 7.5A \Rightarrow I_{rms} = 7.5A$

$Q = V_{rms} \cdot I_{rms} \sin(\varphi_v - \varphi_i)$

$\frac{-1.8\times 10^3}{7.5} = V_{rm} \sin(\varphi_v - \varphi_i)$

F $\quad V_{rms} \sin(\varphi_v - \varphi_i) = -240$

$A = V_{rms} I_{rm}$

$\frac{3.3\times 10^3}{7.5} = V_{rms} \Rightarrow \boxed{V_{rms} = 440V}$

$\sin(\varphi_v - \varphi_i) = \frac{-240}{440}$

$(\varphi_v - \varphi_i) = \sin^{-1}(-0.545)$

$= -33.055^\circ$

$Z = \frac{440}{7.5} e^{-j33.055}$

$$\boxed{Z = 58.66\, e^{-j33.055}\,\Omega}$$

c) $P = 3$ kW $\quad Q = 4$ kVAR $\quad |\hat{V}$

$V_{rms} =$

$P = V_{rms} I_{rms} \cos(\varphi_v - \varphi_i)$

$\frac{3000}{880} = I_{rms} \cos(\varphi_v - \varphi_i)$

$\boxed{I_{rms}\cos(\varphi_v - \varphi_i) = 3.409}$ — ①

$Q = V_{rms} I_{rms} \sin(\varphi_v - \varphi_i)$

$\frac{4000}{880} = I_{rms} \sin(\varphi_v - \varphi_i)$

$\boxed{I_{rms}\sin(\varphi_v - \varphi_i) = 4.545}$

$\tan(\varphi_v - \varphi_i) = 1.33$

$\boxed{\varphi_v - \varphi_i = 53.13^\circ}$ put in ①

$I_{rms} = \frac{3.409}{0.599} \Rightarrow \boxed{I_{rms} = 5.681A}$

$Z = \frac{880}{5.681} e^{j53.13}$

$$\boxed{Z = 154.88\, e^{j53.13^\circ}\,\Omega}$$

d) $|\hat{V}| = 208$ V $\quad |\hat{I}| = 17.8$ A

$V_{rms} = 208$V $\quad I_{rms} = 17.8$ A

$P = V_{rms} I_{rm} \cos(\varphi_v - \varphi_i)$

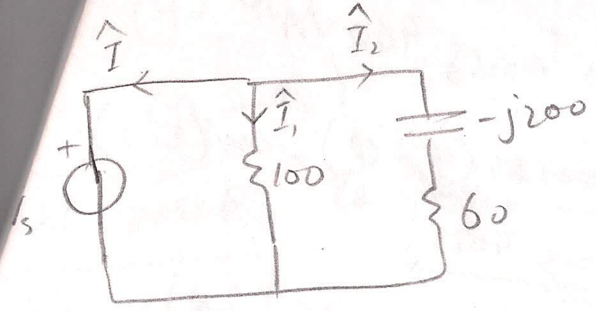
$\cos(\varphi_v - \varphi_i) = \frac{3000}{208 \times 17.8} = 0.81028$

$\varphi_v - \varphi_i = \pm 35.87$

$Z = 11.68\, e^{\pm j35.87}$

$\hat{V}_s = 15\angle 0°$, $R_1 = 100\,\Omega$

$R_2 = 60\,\Omega$ $\quad Z_C = -j200\,\Omega$



$$\hat{I}_2 = \frac{15\angle 0°}{60-j200} = \frac{15\angle 0°}{208.80\angle -73.30}$$

$$\hat{I}_2 = 0.0718\angle 73.30°\ A \checkmark$$

$$\hat{I}_1 = \frac{15\angle 0°}{100} = 0.15\angle 0°\ A \checkmark$$

$$S_2 = \hat{V}_s \hat{I}_2^* = 15e^{j0}\cdot 0.0718\,e^{-j73.30}$$

$$\boxed{S_2 = 1.077\,e^{-j73.30}\ VA}$$

$$S_1 = \hat{V}_s \hat{I}_1^* = 15e^{j0}\cdot 0.15\angle 0°$$

$$\boxed{S_1 = 2.25\ VA}$$

b) $\hat{I} = -(\hat{I}_1 + \hat{I}_2)$

$$= -(0.15 + 0.02063 + j0.06877)$$

$$\hat{I} = -0.17063 - j0.06877$$

$$\hat{I} = 0.1836\angle \cancel{-158.04}\ 21.95 - 180° = -158$$

$$S = \hat{V}_s \hat{I}^* = 15\angle 0° \cdot 0.1839\angle \cancel{158.04}$$

$$\boxed{S = 2.7585\,e^{-j\cancel{158.04}\ 21.95}\ VA}$$

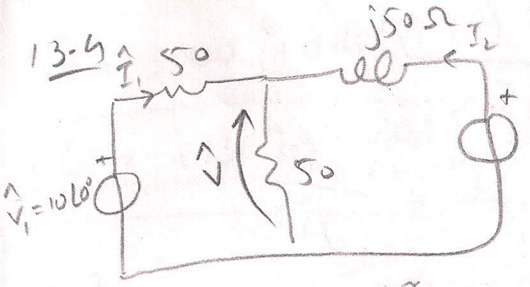
$\cos(\varphi_v - \varphi_i)$

$\cos(158°) = 0.93$

$$PF = \cos(\varphi)$$

$$\boxed{PF = 0.93}$$

13-4



$$\hat{V} = \frac{\frac{j10}{j50} + \frac{10}{50}}{\frac{1}{50} + \frac{1}{50} + \frac{1}{j50}}$$

$$\hat{V} = \frac{\frac{2}{5}}{\frac{2j+1}{j50}} = \frac{20j}{2j+1} \times \frac{1-2j}{1-2j}$$

$$\hat{V} = \frac{20j + 40}{5} = 8 + 4j \checkmark$$

$$\hat{I}_1 = \frac{10 - (8+4j)}{50}$$

$$\hat{I}_1 = \frac{2-4j}{50} = \frac{1-2j}{25}\ A$$

$$\cancel{S_1 = 10}\ \hat{I}_1 = 0.04 - 0.08j\ A$$

$$= 0.089\angle -63.43$$

$$S_1 = 10\angle 0° \cdot 0.089\angle +63.43 \checkmark$$

$$S_1 = 0.89\,e^{j63.43}$$

$$\boxed{S_1 = (0.39 + j0.796)\ VA}$$

$$\cancel{S_2 = \hat{V}_2 \cdot \hat{I}_2^* = \frac{\hat{V}_2 \times \hat{V}_2^*}{-j50}}$$

$$\cancel{S_2 = \frac{10\angle 90 \cdot 10\angle -90}{j50}}$$

$$\cancel{= \frac{100}{j50} \cdot \frac{-j50}{-j50}}$$

$$\cancel{= \frac{-5000j}{2500}}$$

$$\hat{I}_2 = \frac{j10 - (8+4j)}{j50}$$

$$\hat{I}_2 = \frac{-8+6j}{j50} \times \frac{-j50}{-j50}$$

$$\hat{I}_2 = \frac{400j + 300}{2500} = \frac{3+4j}{25}$$

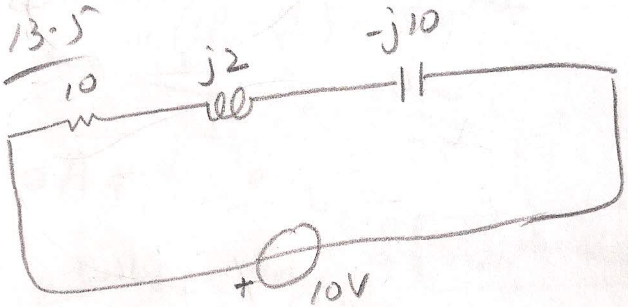
$$\hat{I}_2 = 0.12 + 0.16j$$

$$S_2 = \hat{V}_2 \cdot \hat{I}_2^*$$

$$= j10 \cdot (0.12 - 0.16j)$$

$$\boxed{S_2 = 1.6j + 1.2 \text{ VA}}$$

13.5



$$I_{rms} = \frac{10}{10 + j2 - j10} = \frac{10}{10 - j8} = \frac{5}{5-j4} \text{ A}$$

$$I_{rms} = \frac{5(5+j4)}{25+16} = \frac{25+20j}{41}$$

$$I_{rms} = 0.609 + 0.487j$$

$$= 0.7802 \angle 38.64$$

$$P = V_{rms} \cdot I_{rms} \cos(0 - 38.64)$$

$$= 7.802(0.7810)$$

$$\boxed{P = 6.094 \text{ W}}$$

13.6 $\hat{V} = 80\text{V}$

$\hat{I} = 12\text{A}$

a) $A = V_{rms} I_{rms} = 960 \text{ VA}$

b) $PF = \cos(\varphi_v - \varphi_i) = \cos(25.5)$

$$\boxed{PF = 0.9025}$$

c) $P = V_{rms} I_{rm} \cos(\varphi_v - \varphi_i)$

$$\boxed{P = 866.48 \text{ W}}$$

d) $Q = V_{rms} I_{rms} \sin(\varphi_v - \varphi_i)$

$$= 960(0.4305)$$

$$\boxed{Q = 413.29 \text{ VAR}}$$

e) $Z_{in} = \frac{\hat{V}}{\hat{I}} \angle 25.5$

$$= 6.67 e^{j25.5}$$

$$= 6.67(\cos(25.5) + j\sin(25.5))$$

$$\boxed{Z_{in} = 6.019 + 2.87j \ \Omega}$$

13.7 a) $S = (1000 + j750) \text{ VA}$

$$S = 1250 \angle 36.869$$

$$PF = \cos(36.869)$$

$$\boxed{PF = 0.8}$$

since $\varphi = +ve$
so it is inductive

---

b) $|\hat{V}| = 440\text{V}$, $|Z_L| = 30\Omega$, $P = 3\text{kW}$

$$P = V_{rms} I_{rm} \cos(\varphi_v - \varphi_i)$$

$$I_{rms} = \frac{V_{rms}}{Z_L} \Rightarrow \boxed{I_{rms} = 14.67\text{A}}$$

$$3000 = 440 \times 14.67 \cos(\varphi_v - \varphi_i)$$

$s(\varphi_v - \varphi_i) = 0.4647$

$(\varphi_v - \varphi_i) = \arccos(0.4647)$

$(\varphi_v - \varphi_i) = \pm 62.30$

Not possible to say about load

$\boxed{PF = 0.46}$

c) $A = 10kVA$, $Q = -8kVA$ $P > 0$

$A = V_{rms} \cdot I_{rms}$

$10000 = V_{rms} \cdot I_{rm}$

$Q = V_{rms} \cdot I_{rms} \sin(\varphi_v - \varphi_i)$

$\frac{-8000}{10000} = \sin(\varphi_v - \varphi_i)$

$\varphi_v - \varphi_i = \arcsin(-0.8)$

$\boxed{\varphi_v - \varphi_i = -53.13^\circ}$

capacitive due to -ve angle

$\boxed{PF = 0.6}$

13.8 $V_{rms} = 440V$, $A = 3kVA$

$PF = 0.9$ (inductive)

$I_{rms} = ?$ $P, Q = ?$

$PF = 0.9$

$\varphi_v - \varphi_i = \arccos(0.9)$

$\boxed{\varphi_v - \varphi_i = +25.841}$

$A = V_{rms} \cdot I_{rms} \Rightarrow I_{rms} = \frac{3000}{440}$

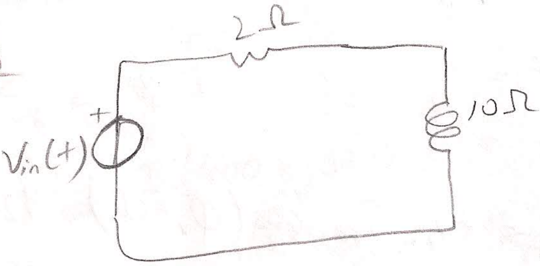
$\boxed{I_{rms} = 6.81A}$

$P = 3000 \times 0.9 \Rightarrow \boxed{P = 2700.02W}$

$Q = 3000 (+0.4358)$

$\boxed{Q = +1307.62 VAR}$

13.9



$P(2) = 8W$ $P(V_{in}) = 8W$

$V_{rms} \cdot I_{rms} \cos(\varphi_v - \varphi_i) = 8W$

$\frac{V_{in}(t)^2}{12} \cos(\varphi_v - \varphi_i) = 8$

$V_{in}(t)^2 \cos(\varphi_v - \varphi_i) = 96$

13.10

$V_{rms} = 2400V$ $A = 10kVA$

$PF = 0.8$ (inductive)

$\boxed{\varphi_v - \varphi_i = 36.86^\circ}$

$\boxed{\frac{10000}{2400} = I_{rms}} \Rightarrow \boxed{I_{rms} = 4.167A}$

$P = 2400 \times 4.167 \cos(36.86)$

$\boxed{P = 8000W}$

$Q = 10000 \sin(36.86)$

$\boxed{Q = 5998.61 VAR}$

$Z = \frac{V_{rms}}{I_{rms}} \angle 36.86$

$\boxed{Z = 575.9539 \Omega}$

13.11 $Q = 4.2\,kVAR$

$V_{rms} = 440V$ $\quad f = 60\,H$

$I_{rms} = 12A$ $\quad \omega = 376.8\ rad/s$

$PF = ?$

$Z = ?$

$\cancel{Z =}\ V_{rms} I_{rms} \sin(\varphi_v - \varphi_i) = 4200$

$\varphi_v - \varphi_i = \arcsin(0.7954)$

$\boxed{\varphi_v - \varphi_i = 3\cancel{7.30}}\ 52.69°$

E

$\boxed{\cancel{Z = 36.67 e^{j37.30°}}}$

$\boxed{\cancel{PF = 0.7954}}$

$\boxed{Z = 36.67 e^{j52.69}\ \Omega \quad PF = 0.6}$

13.12 $I_{rms} = 22A$ $\quad V_{rms} = 220V$

$PF = 0.9$ (inductive)

$\boxed{\varphi_v - \varphi_i = 25.84°}$

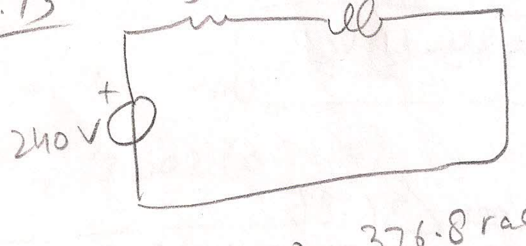
$Z = ?$

$\boxed{Z = 10 e^{j25.84}\ \Omega}$

$PF = 0.8$ $\quad \varphi_v - \varphi_i = 36.869°$

$\boxed{Z = 10 e^{j36.869}\ \Omega}$

13.13



$f = 60Hz$ $\quad \omega = 376.8\ rad/s$

$\hat{I} = \dfrac{240}{50 + \cancel{j0}\ j37.68} \times \dfrac{50 - \cancel{j0}\ j37.68}{50 - \cancel{j0}\ j37.68}$

B

$\hat{I} = \dfrac{12000 - \cancel{24j}\ 9043.2j}{2500 + \cancel{0.01}\ 1419.78} = \dfrac{1200\ldots}{\cancel{250}\ 39\ldots}$

$\cancel{\hat{I} = 0.4799 - 0.009599j}$

$\cancel{\hat{I} = 0.4799 e^{-j1.14588}}$

$\hat{I} = 0.306 - j0.0061227$

$\hat{I} = 0.0936$

$\hat{I} = 3.06 - j2.307$

$\hat{I} = 3.83 \angle -37.01 = 3.83 e^{-j37.01}\ A$

$\cancel{\hat{V} = (3.06 - j2.307)(50 + j37.68)}$

$\hat{V} = 3.83 e^{-j37.01} \cdot 62.608 e^{j37.0016}$

$= 239.78$

$\hat{V}_R = 3.83 e^{-j37.01} \times 50$

$\boxed{\hat{V}_R = 191.5 e^{-j37.01}\ V}$

$\hat{V}_L = (3.83 e^{-j37.01})(j37.68)$

$\hat{V}_L = (3.06 - j2.307)(j37.68)$

$\hat{V}_L = 115.3008j + 86.927$

$\hat{V}_L = 144.397 \angle 52.986$

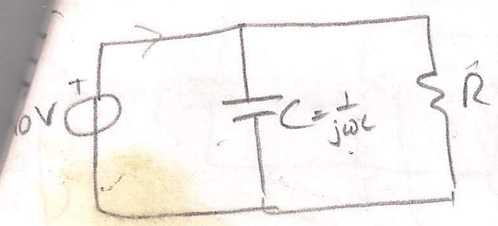
$\boxed{\hat{V}_L = 144.397 e^{j52.986}}$

$S = \hat{V} \cdot \hat{I}^*$

$= 240 \times 3.83 e^{j37.01}$

$\boxed{S = 919 e^{j37.01}\ VA}$

(4)



$= 60\,H \qquad \omega = 376.8\ rad/s$

$$S = (10 - j126)\ VA$$

$$Z_{eq} = \frac{\frac{R}{\cancel{j\omega C}}}{\frac{jR\omega C + 1}{\cancel{j\omega C}}} = \frac{R}{1 + jR\omega C}$$

$$I_{rms} = \frac{440}{\frac{R}{1 + jR\omega C}} = \frac{440(1 + jR\omega C)}{R}$$

$$S = 440 \cdot \frac{440(1 - jR\omega C)}{R}$$

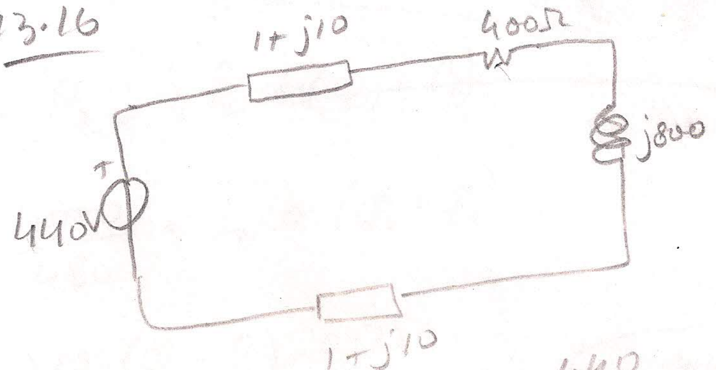
$$10 - j126 = \frac{193600}{R} - j\frac{72948480}{R}RC$$

$$\frac{193600}{R} = 10 \Rightarrow \boxed{R = 19360\,\Omega}$$

$$72948480\,C = 126$$

$$\boxed{C = 1.7\,\mu F}$$

13.16



$$\hat{I} = \frac{440}{2 + j20 + 400 + j800} = \frac{440}{402 + j820}$$

$$\hat{I} = \frac{176880 - 360800j}{834004}$$

$$\hat{I} = 0.212 - 0.4326j$$

a) $\boxed{\hat{I} = 0.48\,e^{-j63.89}\ A}$

b) $S_L = \hat{V}_L \hat{I}^*$

$$= \hat{I}(400 + j800)\hat{I}^*$$

$$= 0.48e^{-j63.89}\ 0.48e^{j63.89}(400 + j800)$$

$$\boxed{S_L = 92.16 + 184.32j\ VA}$$

c) $S_W = \hat{V}_W \hat{I}^*$

$$= \hat{I}(2 + j20)\ 0.48e^{j63.89}$$

$$= 0.2304(2 + j20)$$

$$\boxed{S_W = 0.4608 + j4.608}$$

c) $\eta = \frac{P_L}{P_{gen}} \times 100$

$$= \frac{206\cos(63.43)}{0.48\cos(-63.89)} \times 100$$

$$= \frac{92.1759}{0.2112} \times 100$$

$$\hat{V}_L = (400 + j800)\ 0.48e^{-j63.89}$$

$$= 894.42e^{j63.43}\ 0.48e^{-j63.89}$$

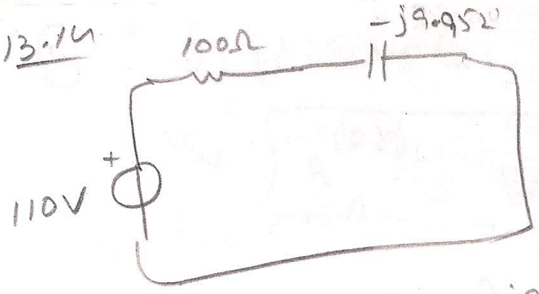
$$\hat{V}_L = 429.32\angle 0^\circ \qquad \hat{I} = 0.48e^{-j63.89}$$

$$\hat{V}_{gen} = 440\angle 0^\circ$$

$$P_L = 206.07\cos(63.89)$$

$$P_{gen} = 440\cos(63.89)$$

13.14



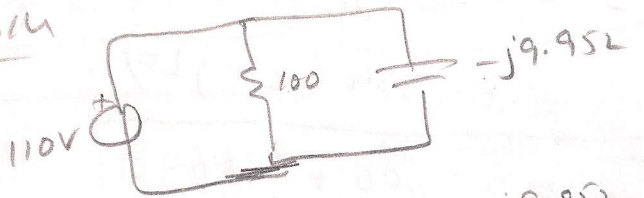
$$\hat{I} = \frac{110}{100 - j9.952} \times \frac{100 + j9.952}{100 + j9.952}$$

$$= \frac{11000 + 1094.745j}{\cancel{1000}\ 10099.0423}$$

$$\hat{I} = 1.089 + 0.1084j$$

$$\boxed{\hat{I} = 1.094\, e^{j5.684}\ A}$$

13.14



$$Z_{eq} = \frac{-j995.2}{100 - j9.952} \times \frac{100 + j9.952}{100 + j9.952}$$

$$= \frac{-99520j + 9904.23j}{10099.0423}$$

$$Z_{eq} = 0.9807 - 9.85j$$

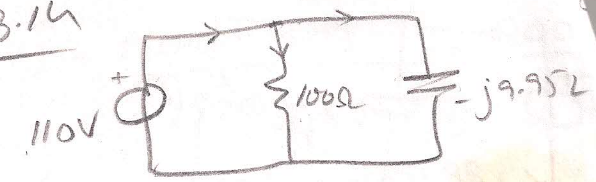
$$\hat{I} = \frac{110}{0.9807 - 9.85j} \times \frac{0.9807 + 9.85j}{0.9807 + 9.85j}$$

$$\hat{I} = \frac{107.877 + 1083.5j}{97.9842}$$

$$= 1.10096 + 11.0578j$$

$$\hat{I} = 11$$

13.14



$$\hat{V}_R = \frac{100}{100 - j9.952} \times 110$$

$$= \frac{11000\,(100 + j9.952)}{10099.0468}$$

$$= \frac{1100000 + 109472j}{10099.0468}$$

$$\hat{V}_R = 108.9211 + 10.839j$$

$$|\hat{V}_R| = 109.45$$

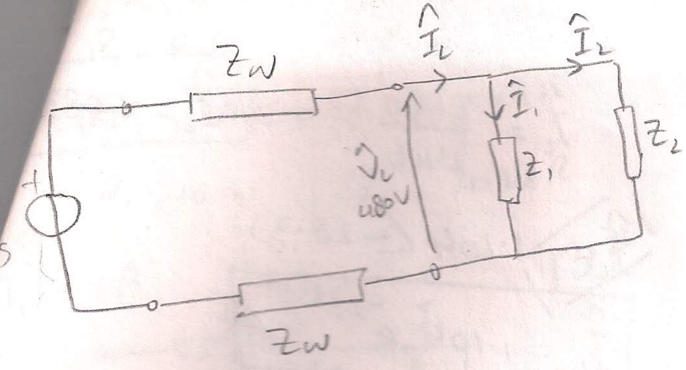
$$\hat{I}_r = \frac{110}{100} = 1.1\ A \checkmark$$

$$\hat{I}_c = \frac{110}{-j9.952} = +j11.05\ A \checkmark$$

$$S = 110\,(1.1 - j11.05) = V \times i^*$$

$$S = 121 - 1215.83j$$

$$\boxed{S = 1221.83\, e^{-j84.31}\ VA}$$



$\hat{V}_L = 480V$

$P(Z_1) = 10kW$ (inductive)

(Z_1) PF $= 0.8$

$P(Z_2) = 12kW$

(Z_2) PF $= 0.75$ (inductive)

$Z_W = 0.35 + j1.5\ \Omega$

$S_s = ?$ $\quad \hat{V}_s = ?$

— $P_{Z_1} = \hat{V}_L \hat{I}_1 \cos(\varphi_v - \varphi_i)$

(Z_1) $\cos(\varphi_v - \varphi_i) = 0.8$

$\varphi_v - \varphi_i = 36.86^\circ \Rightarrow \boxed{\varphi_i = -36.86}$

$\frac{10000}{480} = \hat{I}_1 \cos(36.86)$

$\hat{I}_1 = \frac{20.83}{0.8001} \Rightarrow \boxed{\hat{I}_1 = 26.034A}$

— $P_{Z_2} = \hat{V}_L \hat{I}_2 \cos(\varphi_v - \varphi_i)$

$\frac{12000}{480} = \hat{I}_2 \cos(\varphi_v - \varphi_i)$

(Z_2) $\cos(\varphi_v - \varphi_i) = 0.75$

$\varphi_v - \varphi_i = 41.409^\circ \Rightarrow \boxed{\varphi_{i_2} = -41.409^\circ}$

$\frac{25}{0.75} = \hat{I}_2 \Rightarrow \boxed{\hat{I}_2 = 33.33A}$

$\hat{I}_L = 26.034 e^{-j36.86} + 33.33 e^{-j41.409}$

$= 26.034(\cos(-36.86) - j\sin(36.86)) +$

$33.33(\cos(-41.409) - j\sin(41.409))$

$= 20.829 - j15.616 + 24.997 - j22.045$

$\hat{I}_L = 45.826 - j37.66\ A$

$\boxed{\hat{I}_L = 59.315 e^{-j39.41^\circ} A}$

$Z_1 = \frac{480}{26.034\angle -36.86} \Rightarrow \boxed{Z_1 = 18.437 \angle 36.86^\circ}$

$\boxed{Z_1 = 14.75 + j11.059\ \Omega}$

$Z_2 = \frac{480}{33.33} \angle 41.409 \Rightarrow \boxed{Z_2 = 14.40 \angle 41.409}$

$\boxed{Z_2 = 10.8 + j9.52\ \Omega}$

$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{265.49\angle 78.269}{14.75 + j11.059 + 10.8 + j9.52}$

$Z_{1\parallel 2} = \frac{265.49\angle 78.269}{25.55 + j20.579}$

$= \frac{265.49\angle 78.269}{32.806\angle 38.849}$

$Z_{1\parallel 2} = 8.09\angle 39.42$

$\boxed{Z_{1\parallel 2} = 6.249 + j5.137\ \Omega}$

$\hat{V}_s = (2Z_w + Z_{load})\hat{I}_L$

$\hat{V}_s = (0.7 + 3j + 6.249 + j5.137)\, 59.315 e^{-j39.41}$

$\hat{V}_s = (6.949 + 8.137j)\, 59.315 e^{-j39.41}$

$\hat{V}_s = 10.7 e^{j49.502} \cdot 59.315 e^{-j39.41}$

$\hat{V}_s = 634.67 e^{j10.09}$

$\boxed{\hat{V}_s = 624.85 + 111.19j}$

$S_s = \hat{V}_s \cdot \hat{I}_L^*$

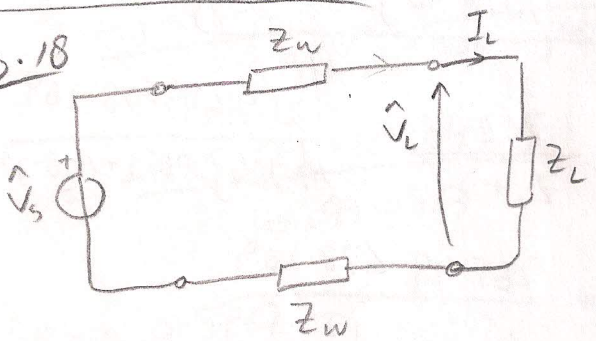
$= 634.67 e^{j10.09} \cdot 59.315 e^{j39.41}$

B

$S_s = 37645.45 e^{j49.5}$

$\boxed{S_s = 24448.76 + 28625.82j}$

13.18



$A(Z_L) = 2.5kVA$

$PF = 0.9 \qquad (\varphi_v - \varphi_i) = 25.84°$

$\varphi_v = 25.84 - 28.35$

$\boxed{\varphi_v = -2.51°}$

$A(\hat{V}_s) = 2.65kVA$

$PF = 0.88 \qquad \varphi_v - \varphi_i = 28.35°$

$\boxed{\varphi_i = -28.35°}$

$Z_L = ?$

$\hat{I}_L = ? \qquad Z_w = ?$

$A(\hat{V}_s) = \hat{V}_s \hat{I}_L$

$\hat{I}_L = \frac{2650}{2400} \Rightarrow \boxed{\hat{I}_L = 1.083}$

$\hat{I}_L = 1.104 \angle -28.35°$

$\boxed{\hat{I}_L = 1.104 e^{-j28.35} A}$

$A(Z_L) = \hat{V}_L \hat{I}_L$

$\hat{V}_L = \frac{2500}{1.104}$

$\boxed{\hat{V}_L = 2264.49 e^{-j2.51} V}$

$Z_L = \frac{2264.49 \angle -2.51}{1.104 \angle -28.35}$

$Z_L = 2051 \angle 25.84$

$\boxed{Z_L = 2051 e^{j25.84} \Omega}$

$\hat{V}_s = \hat{I}_L (2Z_w + Z_L)$

$\frac{2400 \angle 0}{1.104 \angle -28.35} = 2Z_w + 2051 e^{j25.84}$

$2173.9 \angle 28.35 - 2051 \angle 25.84 = 2Z_w$

$Z_w = 1086.95 \angle 28.35 - 1025.5 \angle 25.84$

$Z_w = 956.58 + j516.145 - 922.96 - j446.97$

$Z_w = 33.62 + j69.171$

$\boxed{Z_w = 76.9 \angle 64.07}$

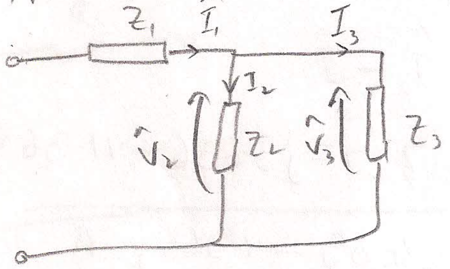
$P_1 = 0\,W\ ,\ Q_1 = -10\,VAR$

$P_2 = 20\,W\ ,\ Q_2 = 10\,VAR$

$P_3 = 10\,W\ ,\ Q_3 = 10\,VAR$

$\boxed{13.20}$

$|\hat{I}_3| = 1A$



$P_3 = \boxed{\hat{V}_3\ \hat{I}_3} \cos(\varphi_v - \varphi_i)$

$\frac{10}{10} = \cos(\varphi_v - \varphi_i)$

$\boxed{\varphi_v - \varphi_i = 90}$

$\boxed{\varphi_v = 90}$

$\hat{V}_3 = \hat{V}_2$

$Z_3 = \frac{\hat{V}_3}{\hat{I}_3} \Rightarrow \boxed{Z_3 = 10\,e^{j90}}$

$\boxed{Z_3 = 10 + j10\ \Omega}$

$Q_3 = \hat{V}_3 \cdot \hat{I}_3 \sin(\varphi_v - \varphi_i)$

$\frac{10}{1} = \hat{V}_3 \sin(90)$

$\boxed{\hat{V}_3 = 10\ V}$

$P_3 + jQ_3 = V_3 I_3^*$

$10 + j10 = V_3$

$Z_3 = 10 + 10j$

$|V_3| \sqrt{200}$

$\hat{V}_3 = \frac{V_3}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}}$

$P_2 + jQ_2 = \hat{V}_2 \cdot \hat{I}_2^*$

$\frac{20 + j10}{10 + j10} = \hat{I}_2^*$

$\hat{I}_2^* = \frac{(20+j10)(10-j10)}{200}$

$\hat{I}_2^* = \frac{200 + 100 + 100j - j200}{200}$

$\hat{I}_2^* = 1.5 - 0.5j$

$\hat{I}_2 = 1.5 + 0.5j$

$Z_2 = \frac{10 + j10}{1.5 + 0.5j} \times \frac{1.5 - 0.5j}{1.5 - 0.5j}$

$= \frac{15 - 5j + 15j + 5}{2.5} = \frac{20 + 10j}{2.5}$

$\boxed{Z_2 = 8 + 4j}$

$\hat{I}_1 = 1.5 + 0.5j + 1$

$\hat{I}_1 = 2.5 + 0.5j$

$P_1 + jQ_1 = \hat{V}_1 \cdot \hat{I}_1^*$

$\frac{0 - j10}{2.5 - 0.5j} = \hat{V}_1 \Rightarrow \hat{V}_1 = \frac{-j10}{2.5 - 0.5j} \times \frac{2.5 + 0.5j}{2.5 + 0.5j}$

$\hat{V}_1 = \frac{-25j - 5}{6.5} = -0.769 - 3.84j$

$\boxed{\hat{I}_1 = -0.769 + 3.84j\ A}$

$\hat{V}_1 = \frac{5 - 25j}{6.5}$

$\boxed{\hat{V}_1 = 0.769 - 3.84j\ V}$

$Z_1 = \frac{0.769 - 3.84j}{2.5 + 0.5j} \times \frac{2.5 - 0.5j}{2.5 - 0.5j}$

$= \frac{1.923 - 0.3845j - 9.6j - 1.92}{6.5}$

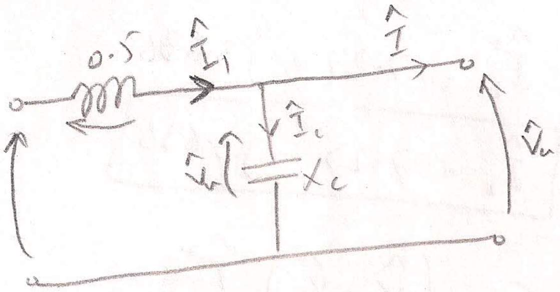
$= 0 - \frac{9.9845}{6.5}j$

$\boxed{Z_1 = -j\,1.53\ \Omega}$

$\underline{13.21}\quad X_C = ?$

$|\hat{V}_i| = 120V \quad |\hat{V}_u| = 100V,\ |\hat{I}| = 10A$

$Q = 600\,VA\ ,\ X_L = 0.5\,\Omega$



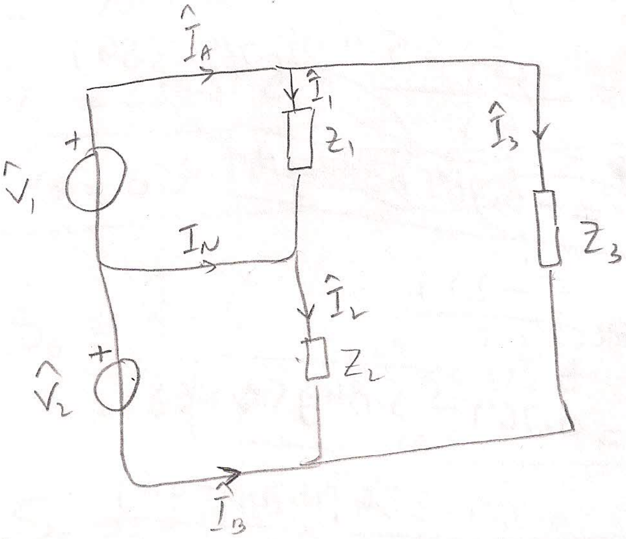
13.19

$S_1 = (1250 + j250)\ VA$

$S_2 = (800 + j400)\ VA$

$S_3 = (2000 + j0)\ VA$

$\hat{V}_1 = \hat{V}_2 = 110\angle 0^\circ V$



$$\boxed{S_1 = \hat{V}_1 \cdot \hat{I}_1^*}$$

$$\frac{1250 + j250}{110} = \hat{I}_1^*$$

$$\hat{I}_1^* = 11.36 + 2.27j$$

$$\boxed{\hat{I}_1 = 11.36 - 2.27j}$$

$$S_2 = \hat{V}_2 \cdot \hat{I}_2^*$$

$$\cancel{2000+}$$
$$800 + 400j = 110\ \hat{I}_2^*$$

$$\hat{I}_2^* = 7.27 + j3.636$$

$$\boxed{\hat{I}_2 = 7.27 - j3.636}$$

$$S_3 = (\hat{V}_1 + \hat{V}_2)\ \hat{I}_3^*$$

$$\hat{I}_3^* = \frac{2000 + j0}{220}$$

$$\boxed{\hat{I}_3 = 9.09\cancel{j} - j0}$$

$$\hat{I}_A = \hat{I}_1 + \hat{I}_3$$

$$= 11.36 - 2.27j + 9.09 - j0$$

$$\boxed{\hat{I}_A = 20.45 - 2.27j}\ A$$

$$\hat{I}_N = \hat{I}_2 - \hat{I}_1$$

$$= 7.27 - j3.636 - 11.36 + 2.27j$$

$$\boxed{\hat{I}_N = -4.09 - 1.366j\ \ A}$$

$$\hat{I}_B = -\hat{I}_2 - \hat{I}_3$$

$$= -7.27 + j3.636 - 9.09$$

$$\boxed{\hat{I}_B = \underset{-16.36}{\cancel{-1.82}} + j3.636\ A}$$

$$S_{V_1} = \hat{V}_1 \cdot \hat{I}_A^*$$

$$= 110\ (20.45 + 2.27j)$$

$$\boxed{S_{V_1} = 2249 + 249.7j\ VA}$$

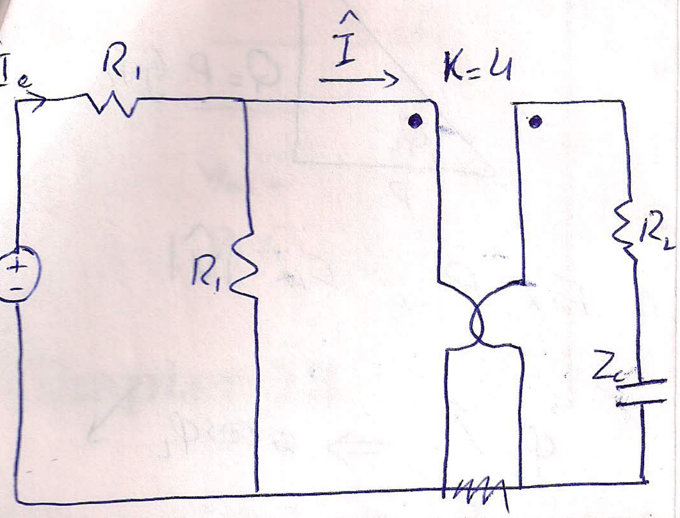
$$\cancel{\alpha}\ S_{V_2} = -\hat{V}_2\ \hat{I}_B^*$$

$$= 110\ (16.36 + j3.636)$$

$$\boxed{S_{V_2} = 1799.6 + 399.96j}$$

$e(t) = 10\cos(\omega t)$ V

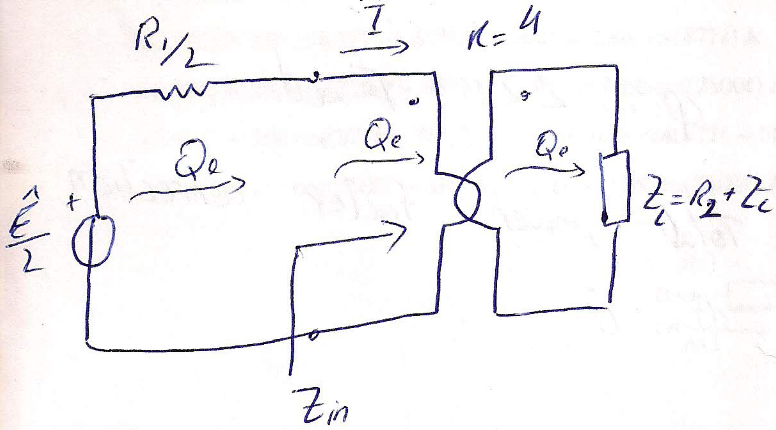
$\boxed{19\text{-}12\text{-}2014}$

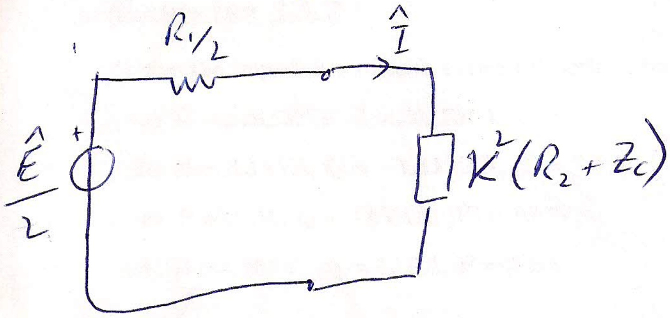


$R_1 = 10\,\Omega$, $R_2 = 5\,\Omega$, $Z_c = -j20\,\Omega$

$I = \frac{E}{R_1} \propto \frac{E}{2}$

$\hat{I}$, $Q = ?$





$$\hat{I} = \frac{\hat{E}/2}{R_1/2 + K^2(R_2 + Z_c)}$$

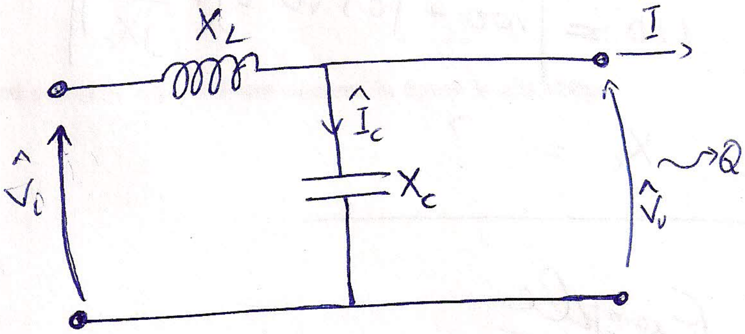
we will calculate power in capacitance only

13.21

Evaluate X_c

$|\hat{V}_i| = 120$ V, $|\hat{V}_u| = 100$ V, $|\hat{I}| = 10$ A,

$Q = 600$ VA, $X_L = 0.5\,\Omega$



$Q = |\hat{V}_u||\hat{I}| \sin\varphi$

$\varphi = \arcsin \frac{Q}{|\hat{V}_u| \cdot |\hat{I}|} = \arcsin \frac{600}{1000}$

$= \arcsin 0.6$

$\varphi = 37°$ ✓

↳ Phase shift

$\varphi = \angle\hat{V}_u - \angle\hat{I} \Rightarrow \angle i = \angle v - \varphi = -37°$

we

$\rightarrow \angle\hat{V}_u = 0$ (reference phase)

$\Rightarrow \hat{I} = 10e^{-j\varphi}$ $10(\cos 37 - j\sin 37)$

$\rightarrow \hat{V}_u = 100$ V $\Rightarrow \hat{I} = (8 - j6)$ A

$$\left(\hat{I} = 10(\cos\varphi - j\underbrace{\sin\varphi}_{0.6})\right)$$

$$\hat{I}_c = \frac{\hat{V}_u}{jX_c} = \frac{100}{jX_c} \quad , X_c < 0 \quad \text{(reactive)}$$

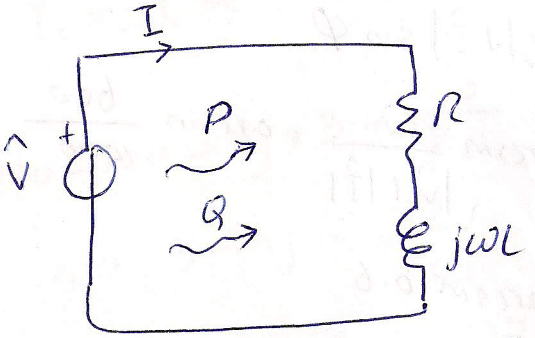
$$\hat{V}_i = \hat{V}_u + j0.5(\hat{I} + \hat{I}_c)$$

$$|\hat{V}_i| = \left|100 + j0.5\left(8-6j + \frac{100}{jX_c}\right)\right|$$

$$120 = \left|100 + j0.5\left(8-6j + \frac{100}{jX_c}\right)\right|$$

$$X_c = ?$$

Example



Find P, Q As functions of $P.F = \cos\varphi$

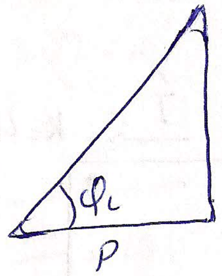
$$\varphi_L = \angle Z_L = \arctan\left(\frac{\omega L}{R}\right)$$

$$Z_L = R + j\omega L$$

$$P = |\hat{V}||\hat{I}|\cos\varphi_L$$

$$Q = |\hat{V}||\hat{I}|\sin\varphi_L$$

$$|\hat{I}| = \frac{P}{|\hat{V}|\cdot\cos\varphi_L}$$



$$Q = P\cdot \tan\varphi_L$$

Fix P , Fix $|\hat{V}|$

$$\varphi_L \nearrow \Rightarrow \cos\varphi_L \searrow$$

$$|\hat{I}| \nearrow$$

$$\varphi_L < \varphi_{max}$$

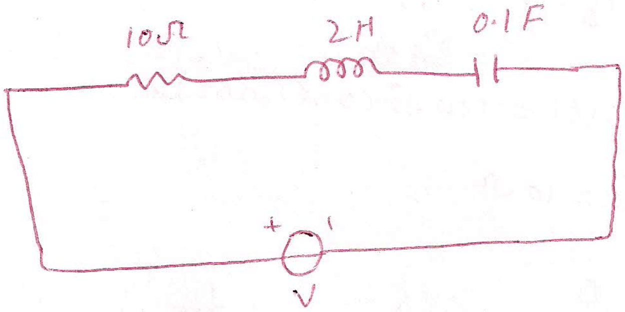
$$\cos\varphi_L > PF_{min}$$

$\varphi_{in} = \angle Z_{in}$ Fixed

Total power factor correction

$$\varphi_{in} = 0^\circ.$$

13.05



$V_{rms} = 10\,V$
$\omega = 1\,rad/s$

Average Power supplied by source?

Solution

Suppose $\angle V = 0^\circ \Rightarrow \hat{V} = 10\sqrt{2}\ V$

So

$$Z_{eq} = 10 + 2j - \frac{j}{0.1}$$

$$= 10 - 8j$$

$$\hat{I} = \frac{\hat{V}}{10 - 8j} = 1.10\angle 38.66^\circ$$

$$\text{Average Power} = \frac{1}{2}|\hat{V}||\hat{I}|\cos(\theta_v - \theta_i)\ W$$

$$= \frac{1}{2}(10\sqrt{2})(1.10)\cos(38.66^\circ)$$

$$= \cancel{5.52}\ 6.07\ W$$

{ ANSWER NOT VERIFIED! But lets try to figure-out the problem. ☺

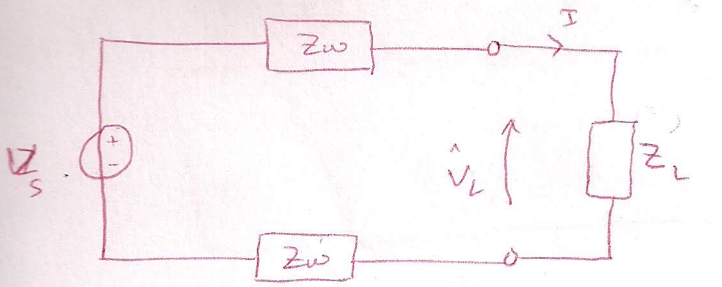
if we take $|\hat{V}| = 10\,V$

$$\hat{I} = 0.788\angle 38.66^\circ$$

$$P = 3.048\ W \Longrightarrow$$

{ Answer VERIFIED if we ignore (rms) factor and treat normally. Yes (PROF. VERIFIED)

13-18



(MY Answer is Right)

$A_{Z_L} = 2.5$ KVA $\Rightarrow$ P.F $= 0.9$ $\quad \theta_1 = 25.84$

$V_s(\text{rms}) = 2400$ v $\Rightarrow$ P.F $= 0.88$, $A_s = 2.65$ KVA $\quad \theta_2 = 28.36°$

$\hat{I}_L, \; Z_L, \; Z_w \Rightarrow ?$

Solution

i) $|\hat{V}_s| = \sqrt{2} \times 2400 = 3394.11$ V

$A_s = \frac{1}{2}|\hat{V}_s||\hat{I}_s|$

$|\hat{I}_s| = \dfrac{2 \times 2.65 \times 10^3}{3394.11} = 1.56$ A

#ERROR
G-A $= 2.21e^{-j28.36°}$

By taking $\angle V_s = 0°$

$\angle V_s - \angle I_s = 28.36$, $\angle i_s = -28.36 \Rightarrow \hat{I}_s = \left(1.56e^{-j28.36°}\right)$ A

2) we Know

$S = Ae^{j\theta}$ VA

So

$S_s = 2.65e^{j28.36}$ KVA

$S_L = 2.5e^{j25.84°}$ KVA

where

$S_w + S_z = S_s$

S_z (complex Power of $2Z_w$)

$= 2.65e^{j28.36} - 2.5e^{j25.84}$ KVA

KVA$\left(2.5e^{j25.84°}\right) = \frac{1}{2}(Z_L)|I_s|^2$

$Z_L = \dfrac{5\angle 25.84°}{2.43}$ KΩ

$Z_L = 2.05\angle 25.84°$ KΩ

#ERROR
G-A $= 1.02\angle 25.84$ KΩ

iii) $\text{KVA}\left(2.65e^{j28.35^\circ} - 2.5e^{j25.84^\circ}\right) = \frac{1}{2} Z_w' |I_s|^2$

$$\text{K}\Omega\left(\frac{2\cdot(2.33+j1.259-2.25-1.09j)}{|I_s|^2}\right) = Z_w'$$

$$S = Ae^{j\theta}$$

$$S = \frac{1}{2} I Z (I)^*$$

$$= \frac{1}{2}|I|^2 Z$$

$$\text{K}\Omega\left(\frac{0.16+0.32j}{|I_s|^2}\right) = Z_w'$$

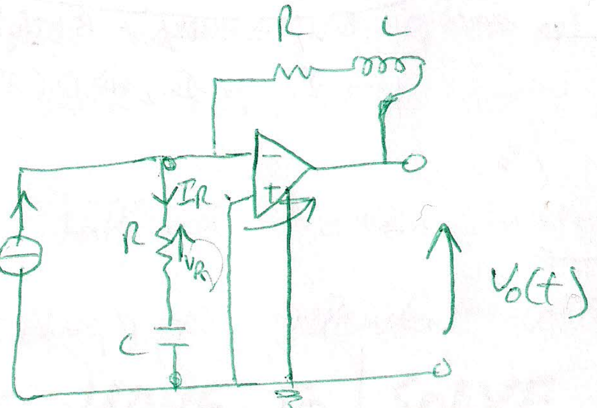
$$Z_w' = \left(\frac{0.35}{|I_s|^2}\angle 63.43^\circ\right)\text{K}\Omega$$

\# ERROR

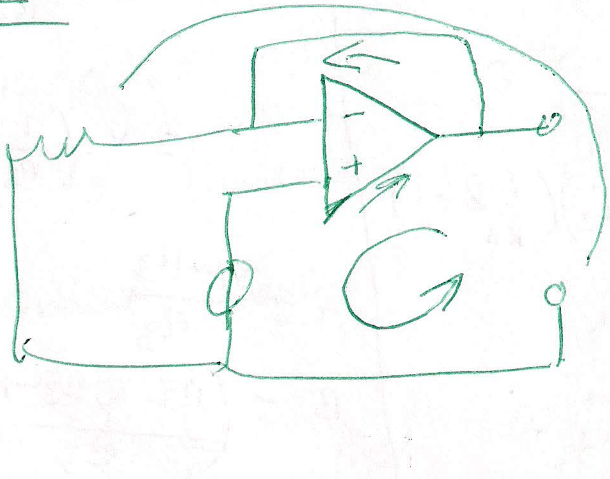
$G\text{-}A = 38.45e^{j64.11^\circ}\ \Omega$

$$Z_w' = 143.81e^{j63.43^\circ}$$

$$Z_w = \frac{143.81}{2}e^{j63.43^\circ} = \left(71.9e^{j63.43^\circ}\right)\Omega$$

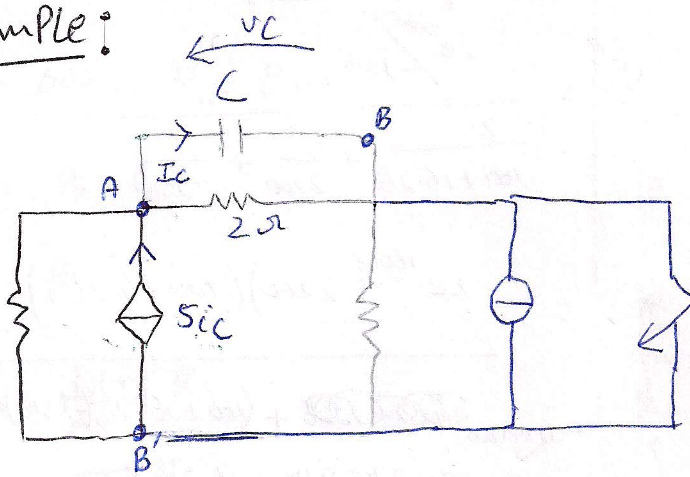


$V_R = ?$, $I_R = ?$



// WHAT happened to Dependent Source when we find initial conditions? Any specific/special thing to keep in mind for that?

EXAMPLE:



i) For $t \to 0^-$ or $t<0$, $i_c = 0$ AB open ⟹ AB' also open? ✓

OR

We just assume AB' open in case of current source and short for voltage source. (Dependent ones) ✗

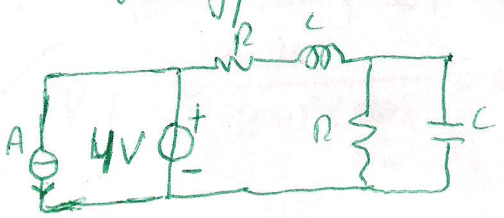
ii) WHAT happened when instead of D.current source we have a dependent voltage source. (A–B' 2vc)

WE HAVE TO SOLVE it AS "CH#4"

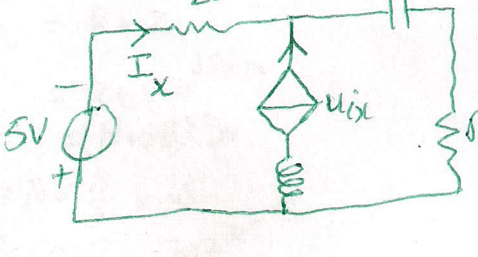
// Transforming from real circuit (time domain) to symbolic circuit (Laplace or frequency domain), what happens to constant value of independent/dependent voltage/current source?

$4V \to \frac{4}{s}$

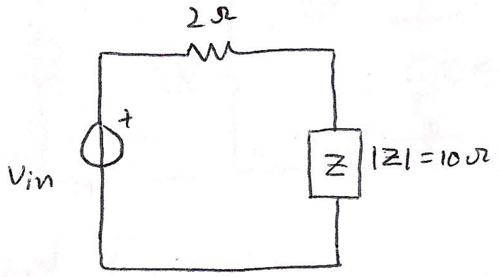
$4i_x(t) \Downarrow 4i_x(s)$



And



13.9



$P_Z = 6w, \quad P_{in} = 8w$

$P_{in} = P_R + P_Z$

$P_R = 8 - 6 = 2w$

$P_R = \frac{1}{2}|\hat{I}|^2 R$

a) $|\hat{I}| = \sqrt{\frac{2 \times 2}{2}} = \sqrt{2} = 1.41 A$

b) Power Factor of Z,

$P_Z = \frac{1}{2}|V_Z||I_Z| \overbrace{\cos\theta}^{P.F \text{ of } Z}$ $\quad \}\ |V_Z| = |I_Z||Z|$

$6 = \frac{1}{2}|I_Z|^2 Z \cos\theta$

$\cos\theta = \frac{12}{10} \div (1.41)^2$

$P.F = 0.6$

c) P.F of Load Seen From Source Terminal:-

Two - METhods

$P_{in} = \frac{1}{2}|\hat{V}_{in}||\hat{I}| \cos\theta$

$\cos\theta = \frac{2 \times P_{in}}{|\hat{I}|^2 |Z_{eq}|}$

$P.F = \frac{16}{(1.41)^2 (8\sqrt{2})} = 0.71$

$\}\ |V_{in}| = |\hat{I}||2+10|$

$\angle Z = \cos^{-1} 0.6$

$\angle Z = 53.13°$

$Z_{eq} = R + Z$

$= 2 + 10e^{j53.12}$

$= (8 + j8)\ \Omega$

$\theta = \tan^{-1}\frac{8}{8} = 45$

$P.F = \cos\theta = .707$

D) Let $\angle I = 0$, Then $\angle Z_{eq} = \angle V - \angle I$

$\angle V = 45°$

$V_{in}(t) = \hat{I} \cdot \hat{Z}_{eq}$

$= (15.95 \angle 45°)\ V$

E) $V_Z(t) = \hat{I} Z$

$= (1.41)(10 \angle 53.13)$

$= (14.1 \angle 53.13°)\ V$

if we Let $\angle V = 0$, then $\angle I = -45°$

verified

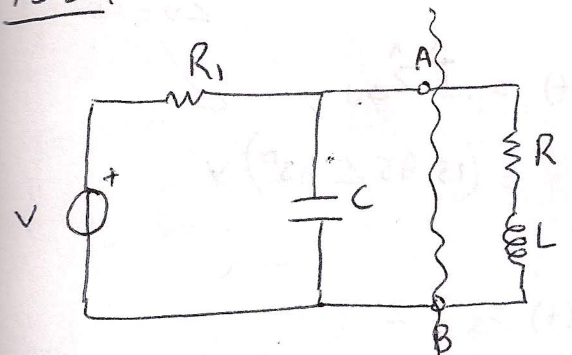
$V_{in}(t) = 16 \cos(\omega t)\ V$

$V_Z(t) = 14.1 e^{j(53.13-45)}$

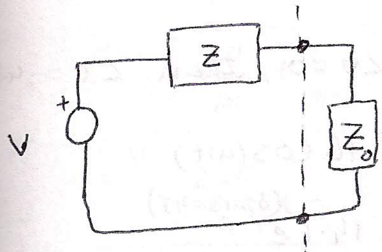
$= 14.1 e^{j8.13°}$

$V_Z(t) = 14.1 \cos(\omega t + 8.13°)\ V$

13.24



FOR MAX-POWER OutPut



$Z_C = \frac{-j10^6}{40\times 2\pi\times 10^3}$

$= -j39.74\ \Omega$

$Z_o = Z^* \Rightarrow Z_o = R + j\omega L$

So $Z = R_1 // Z_C$

$= \frac{(3)(-j3.97)}{3-j3.97}$

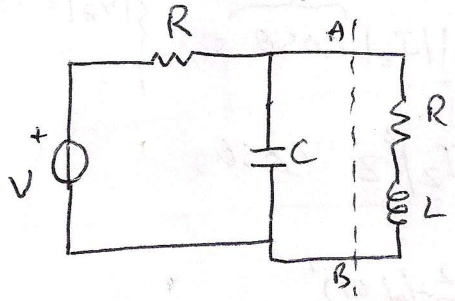
$Z_o = Z^* = \frac{j12}{3+j3.97}\times\frac{3-j3.97}{3-j3.97}$

$\boxed{R = \frac{48}{25} = 1.92\ \Omega}$

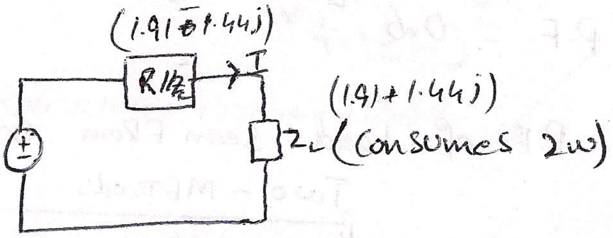
$\cancel{j}\omega L = \frac{36\cancel{j}}{25} = 1.44$

$\boxed{L = \frac{1.44}{2\pi f} = 0.229\ mH}$

Now we Know that under PoweR matching Conditions Zo consumes 2w active PoweR. and we have to find $|\hat{V}|$.



Thevnin equivilent says that (AB)



$V_{eq} = V\left(\frac{Z_C}{R_1+Z_C}\right) \Leftarrow V_{eq}$

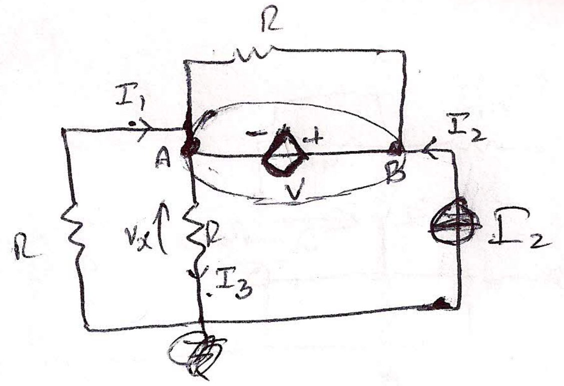
$\hat{I} = \frac{V_{eq}}{3.82}$

$P_s = \frac{1}{2}\frac{V_{eq}}{3.82}\cdot(1.91+1.44j)\cdot\frac{(V_{eq})^*}{3.82}$

$2 = \frac{1.91|V_{eq}|^2}{2\times 3.82\times 3.82} \Rightarrow V_{eq} = 5.528V$

$\left|\frac{R_1+Z_C}{Z_C}\right| |5.528| = |\hat{V}|$

$|\hat{V}| = 6.92V$



$I_1 - I_3 + I_2 = 0$

Taking AB AS Supelnode,
what will be KCL :-

we will ignore the BRanch cullent which is Paralel to SUPER-NODE

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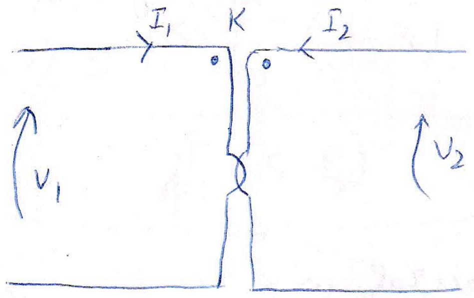
i) FoR Req of a CKT (Ch#9) Plefel Test soulce Method.

ii) You will find this Ratio in Exam (ALMOST)

CH	no.Q
1 – 4	2
6	1
9	1 Shost (MAY Be one Big Question ALSO)
10	1 Short
11	Big Question
12	1 Short (MAY Be Big ALSO)
13	1
14	1 Short

You must have complete grip on (KCL, KVL, MILLMAN Thevnon/NORTON)

TRASFORMER:-

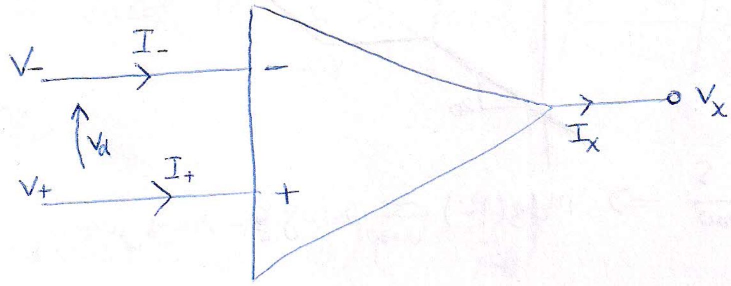


$V_1 = K V_2$ ——— (i)

$I_1 = -\frac{1}{k} I_2$ ——— (ii)

ALWAYS

AMPLIFIER:-



$V_- = V_+$, SO $V_d = 0$ | ALSO $I_- = I_+ = 0$

BUT $\Longrightarrow$ V_x, I_x ARe Unknown veRiABLeS,